PART – A

PHYSICS

1.

Ans. [1]

Sol. Let volume of man is Lbh

As all dimension increases by a factor (K = 9) keeping the density constant

Stress on his legs = $\frac{\text{weight}}{\text{area}} = \frac{V\rho g}{A}$

Initial stress = $\text{Stress}_1 = \frac{V\rho g}{A}$

Final stress = $\text{Stress}_2 = \frac{K^3 V\rho g}{K^2 A}$

$\boxed{\text{Stress}_2 = K\ \text{Stress}_1}$

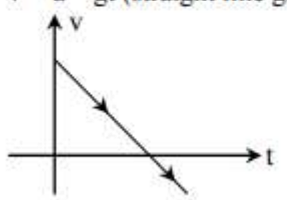
Where (K = 9)

So stress is changed by a factor 9.

2.

Ans. [3]

Sol. v = u – gt (straight line graph)



Ans. [3]

Sol. $a = -\frac{kv^2}{m}$

$\frac{dv}{dt} = -\frac{kv^2}{m}$

$\int_{10}^{v} \frac{dv}{v^2} = -\frac{k}{m}\int_{0}^{10} dt$

$\left[-\frac{1}{v}\right]_{10}^{v} = -\frac{k}{m} \times 10$

$-\left[\frac{1}{v} - \frac{1}{10}\right] = \frac{k}{10^{-2}} \times 10$

$-\frac{1}{v} + \frac{1}{10} = -k \times 1000$

According to question

$KE = \frac{1}{2}mv^2 = \frac{1}{8}mv_0^2$

$v = \frac{v_0}{2} = \frac{10}{2}$

$-\frac{1}{10} \times 2 + \frac{1}{10} = -k \times 1000$

$\frac{1}{10} = k \times 1000$

$k = 10^{-4}$

Ans. [1]

Sol. $a = \frac{6t}{1} = 6\,t$

$\frac{dv}{dt} = 6t$

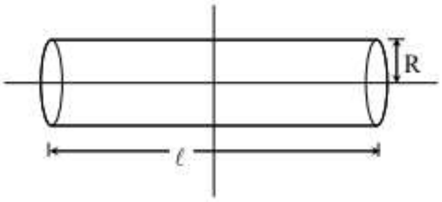
$v = \left[\frac{6t^2}{2}\right]_0^1 = 3 \times 1^2 = 3$

$KE = \frac{1}{2} \times 1 \times 3^2 = 4.5$

$W = \Delta KE = 4.5 - 0 = 4.5$ Joule

Ans. **[1]**

Sol.



Moment of inertia of cylinder about perpendicular bisector is I

$$I = M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$$

For given mass and density

$$M = \pi R^2 L\rho$$

$$R^2 = \frac{M}{\pi L\rho}$$

$$I = M\left[\frac{L^2}{12} + \frac{M}{4\pi L\rho}\right]$$

For maxima or minima of I

$$\frac{dI}{dL} = 0$$

$$\frac{dI}{dL} = M\left[\frac{2L}{12} - \frac{M}{4\pi L^2\rho}\right] = 0$$

$$\frac{L}{6} = \frac{M}{4\pi L^2\rho}$$

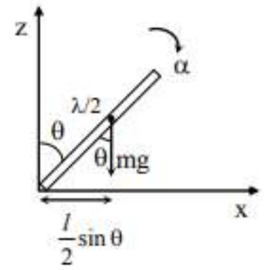
$$\frac{L}{6} = \frac{\pi R^2 L\rho}{4\pi L^2\rho}$$

$$\frac{L^2}{R^2} = \frac{3}{2}$$

$$\boxed{\frac{L}{R} = \sqrt{\frac{3}{2}}}$$

Ans. [1]

Sol.



$\tau = mg\frac{l}{2}\sin\theta$

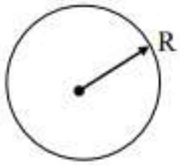
$\tau = I\alpha$

$mg\frac{l}{2}\sin\theta = \frac{ml^2}{3}\alpha$

$\alpha = \frac{3}{2}\frac{g}{l}\sin\theta$

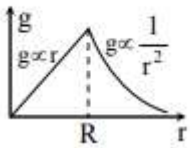
Ans. [4]

Sol.



$g = \frac{GM}{r^2}$ when $r > R$

$g = \frac{GMr}{R^3}$ when $r < R$



Ans. [2]

Sol. Total heat gain = Total heat loss

$100 \times 0.1\,(75-30) + 170 \times 1 \times (75-30) = 100 \times 0.1\,(T-75)$

$10 \times 45 + 170 \times 45 = 10T - 750$

$1200 + 7650 = 10\,T$

$T = 885°C$

Ans. [1]

Sol.

$$B = \frac{-P}{\frac{dV}{V}}$$

$$\frac{dV}{V} = \frac{-P}{B}$$

$$dV = \frac{-PV}{B}$$

By heating we have to increase the volume by $\frac{PV}{B}$

$$\Delta V = V\gamma\Delta T = V \times 3\alpha\ \Delta T$$

$$V \times 3\alpha\Delta T = \frac{PV}{B}$$

$$\Delta T = \frac{P}{3\alpha B}$$

Here B = K

$$\therefore \Delta T = \frac{P}{3\alpha B}$$

Ans. [3]

Sol. $C_p - C_v = R$

If C_p and C_v are molar specific heat

But if C_p and C_v are specific heat i.e. gram specific heat then

$C = MS_g$ $\{Q = 1C\Delta T \Rightarrow Q = MS_g\Delta T \Rightarrow C = MS_g\}$

$$S_g = \frac{C}{M}$$

$$MS_{gp} - MS_{gv} = R$$

$$S_{gp} - S_{gv} = \frac{R}{M}$$

$$\frac{R}{2} = a$$

$$\frac{R}{28} = b$$

$$14 = \frac{a}{b}$$

$$a = 14\ b$$

Ans. **[4]**

Sol. $PV = nRT$

$$n = \frac{PV}{RT}$$

$T_i = 273 + 17$

$= 290$ K

$T_f = 273 + 27$

$= 300$ K

$$n_f - n_i = \frac{10^5 \times 30}{8.314}\left[\frac{1}{300} - \frac{1}{290}\right] \times 6.023 \times 10^{23}$$

$$= \frac{3\times10^6}{8.314}\left[\frac{-10}{300\times290}\right] \times 6.023 \times 10^{23}$$

$$= -\frac{3\times10^{27}\times6.023}{8.314\times3\times29}$$

$$= -\frac{-6.023\times10^{27}}{8.314\times29}$$

$$= -0.025 \times 10^{27}$$

$$= -2.5 \times 10^{25}$$

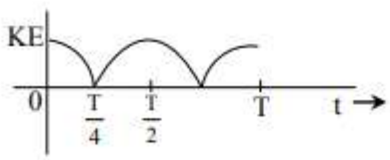
Ans. **[4]**

Sol. $x = A \sin \omega t$

$$v = \frac{dx}{dt} = A\omega \cos \omega t$$

$$KE = \frac{1}{2} mA^2\omega^2\cos^2\omega t$$

$$= \frac{1}{2} mA^2\omega^2\cos^2\frac{2\pi}{T}t$$



Ans. [3]

Sol. According to theory of relativity

$$f_{app} = \left[\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right] f \quad \text{(for approach)}$$

$$= \frac{1+\frac{c/2}{c}}{\sqrt{1-\left(\frac{c/2}{c}\right)^2}} \times 10\text{ GHz}$$

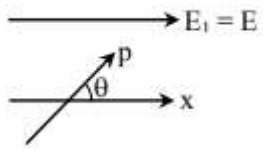
$$= \frac{\frac{3}{2}}{\sqrt{\frac{3}{4}}} \times 10\text{ GHz}$$

$$= \sqrt{3} \times 10\text{ GHz}$$

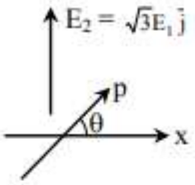
$$f_{app} = 17.32\text{ GHz}$$

Ans. [3]

Sol.



$$\tau = \vec{T}_1 = PE\sin\theta\ \hat{k}$$



$$\tau = \vec{T}_2 = P\sqrt{3}E\cos\theta(-\hat{k})$$

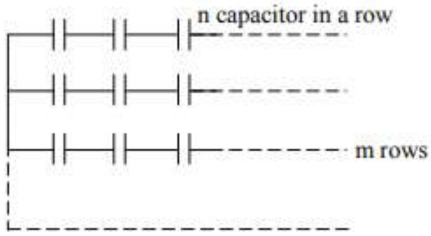
$$PE\sin\theta\,(\hat{k}) = -P\sqrt{3}E\cos\theta(-\hat{k})$$

$$PE\sin\theta = \sqrt{3}E\cos\theta$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

Ans. **[4]**
Sol.



Potential on each capacitor $V = \frac{1000}{n}$

$\frac{1000}{n} = 300$

$n = \frac{10}{3} \approx 4$

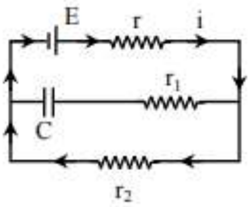
$C_{eq} = \frac{C}{n} \times m$

$\frac{1}{n} \times m = 2$

$m = n \times 2 = 4 \times 2 = 8$

Minimum number of capacitor $= 8 \times 4 = 32$

Ans. **[3]**
Sol.



At steady state current through capacitor branch become zero.

$i = \frac{E}{r + r_2}$

Potential difference across capacitor ΔV

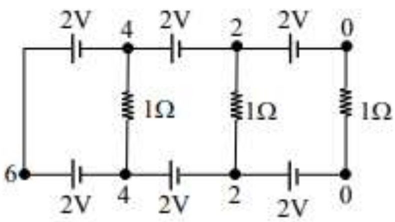
$\Delta V = ir_2$

$\Delta V = \left(\frac{E}{r + r_2}\right) r_2$

charge on capacitor $= C\,\Delta V$

$= CE\left(\frac{r_2}{r + r_2}\right)$

Sol.



Potential difference across each resistor is zero so current in each resistor also zero.

Ans. [1]

Sol. $|\vec{M}| = 6.7 \times 10^{-2}\ Am^2$

$I = 7.5 \times 10^{-6}$ kg m^2, B = 0.01 T

$\tau = -MB\sin\theta$

$I\alpha = -MB\theta$ (for small oscillations)

$\alpha = \left(\frac{MB}{I}\right)\theta \quad \Rightarrow \omega = \sqrt{\frac{MB}{I}} \Rightarrow T = 2\pi\sqrt{\frac{I}{MB}}$

$T = 2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}}$

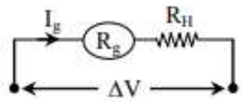
$\Rightarrow T = 0.6644$ sec

Time for 10 oscillation $\Delta t = 10\,T \Rightarrow \Delta t = 6.65$ sec

Ans. [1]

Sol. $I_{g\,max} = 5$ mA , $R_g = 15\ \Omega$

Range of voltmeter = 10 volt



$\Delta V = I_g(R_g + R_H)$

Range $\Delta V_{max} = I_{g\,max}(R_g + R_H)$

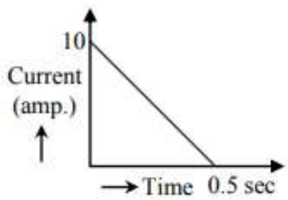
$10 = 5 \times 10^{-3}(15 + R_H)$

$R_H = 1985\ \Omega$

$R_H = 1.985 \times 10^3\ \Omega$

Ans. [3]

Sol.



Emf = iR

$-\frac{d\phi}{dt} = iR$

$\int(-d\phi) = \int(idt)\ R$

$(-\Delta\phi) = R\int_0^{0.5} idt$

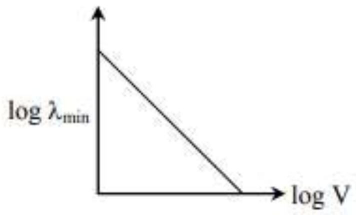
$|(-\Delta\phi)| = R$ (area of i – t curve)

$\Delta\phi = R\left(\frac{1}{2}\times0.5\times10\right)$

$\Delta\phi = 100\ (2.5)$

$\Delta\phi = 250$ Wb

Ans. [1]
Sol.



K.E. = eV (K.E. = kinetic energy of electron)
$E_{P_{max}}$ = K.E.

$\frac{hc}{\lambda_{min}} = eV \Rightarrow \lambda_{min} V = \left(\frac{hc}{e}\right)$ = constant

$\ell n\, \lambda_{min} + \ell n\, V = \ell n$ constant

$\ell n\, \lambda_{min} = -\ell n\, V + \ell n$ (constant)

Straight line of – ve slope

Ans. [1]
Sol.

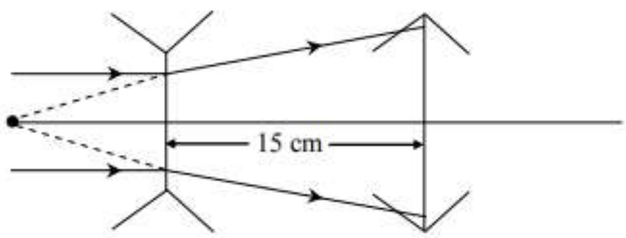


Image form by diverging is at the focus of diverging lens.
Now image form by diverging act as a source for converging lens.
For converging lens object real at a distance 40 cm from it which is at ($2f$).

$u = -2f, f = +f, v = +2f$ $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$

Final image real at a distance $2f$ = 40 cm from converging lens.

Ans. [2]
Sol. d = 0.5 mm, D = 1.5 m, λ_1 = 650 nm, λ_2 = 520 nm

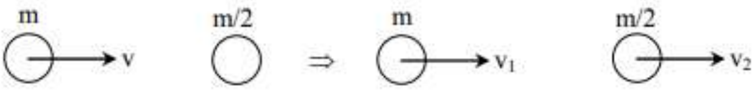
$\beta_1 = \frac{\lambda_1 D}{d} = \frac{650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$ = 1.95 mm

$\beta_2 = \frac{\lambda_2 D}{d} = \frac{520 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$ = 1.56 mm

Least distance where their maxima again coincides from central maxima is = LCM of β_1 & β_2 = 7.8 mm

Ans. [2]

Sol.



$$v_1 = \frac{m - m/2}{m + m/2}v + 0$$

$$v_1 = \frac{m/2}{3m/2}v$$

$$v_1 = \frac{v}{3}$$

Similarly

$$v_2 = 0 + \frac{2m}{m + m/2} \times v$$

$$v_2 = \frac{2m}{3m/2}v$$

$$v_2 = \frac{4}{3}v$$

$$\because \lambda_1 = \frac{h}{m_1 v_1}, \lambda_2 = \frac{h}{m_2 v_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2 v_2}{m_1 v_1} = \frac{m/2}{m} \cdot \frac{4v/3}{v/3} = 2$$

Ans. [4]

Sol.

$$\frac{hc}{\lambda_1} = (-E) - (-2E)$$

$$\frac{hc}{\lambda_1} = E \qquad \text{....(i)}$$

$$\frac{hc}{\lambda_2} = (-E) - \left(-\frac{4E}{3}\right) = -E + \frac{4E}{3} = \frac{-3E + 4E}{3} = \frac{E}{3} \qquad \text{....(ii)}$$

By (i) & (ii)

$$r = \frac{\lambda_1}{\lambda_2} = \frac{\frac{hc}{E}}{\frac{hc}{E/3}} = \frac{1}{3}$$

Ans. [2]

Sol. $A \longrightarrow B$

$$A = A_0 e^{-\lambda t};\ B = A_0(1 - e^{-\lambda t});\ \frac{B}{A} = \frac{A_0(1 - e^{-\lambda t})}{A_0 e^{-\lambda t}}$$

$$0.3 = e^{\lambda t} - 1$$

$$e^{\lambda t} = 1.3$$

$$\lambda t = \ln(1.3)$$

$$\frac{\ln(2)}{T}t = \ln(1.3)$$

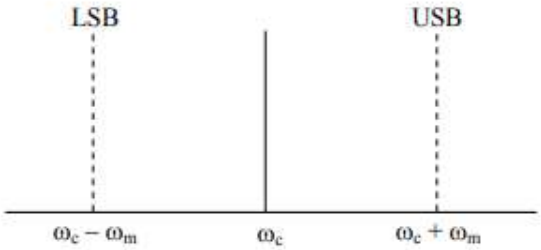
$$t = T\,\frac{\ln(1.3)}{\ln(2)} \Rightarrow t = \frac{T\log(1.3)}{\log(2)}$$

Ans. [4]

Sol. In C-E amplifier phase difference between input-output voltage is 180°.

Ans. [1]

Sol.

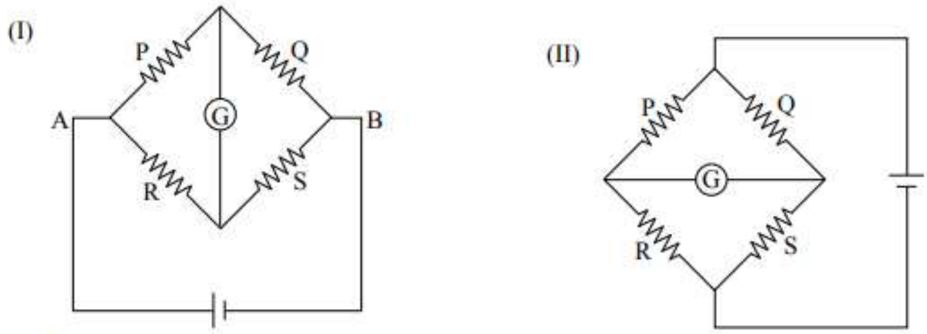


because $\Delta\omega_m << \omega_c$

$\therefore$ ω_m is not present in modulated wave.

Ans. [2]

Sol.



In I[st] case for balance

$$\frac{P}{R} = \frac{Q}{S}$$

$$\frac{P}{Q} = \frac{R}{S} \quad \text{....(i)}$$

In II[nd] case for balance

$$\frac{P}{Q} = \frac{R}{S} \quad \text{....(ii)}$$

In both case

Null point is same.

Ans. [2]

Sol. $D = 1.25 \times 10^{-2}$ m, $\Delta D = 0.01 \times 10^{-2}$ m, $h = 1.45 \times 10^{-2}$ m, $\Delta h = 0.01 \times 10^{-2}$ m

$g = 9.80\ m/s^2$

$$T = \frac{rhg}{2} \times 10^3\ N/m$$

$$Tr^{-1}h^{-1} = \frac{g}{2} \times 10^3$$

$Tr^{-1}h^{-1}$ = constant

$$\frac{\Delta T}{T} - \frac{\Delta r}{r} - \frac{\Delta h}{h} = 0$$

$$\frac{\Delta T}{T} = \left(\frac{\Delta r}{r} + \frac{\Delta h}{h}\right)$$

$$\frac{\Delta T}{T} = \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}}\right) + \left(\frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}}\right) \left\{\begin{matrix} D = 2r \\ \Delta D = 2\Delta r \\ \frac{\Delta D}{D} = \frac{\Delta r}{r} \end{matrix}\right\}$$

$$\% \text{ error} = \frac{\Delta T}{T} \times 100$$

$\approx 1.5\ \%$

CHEMISTRY

Ans. [1]

Sol.

$C_{(graphite)} + O_2(g) \longrightarrow CO_2(g)$; $\Delta_r H° = -393.5$ kJ .... (i)

$H_2(g) + \frac{1}{2} O_2(g) \longrightarrow H_2O(1)$; $\Delta_r H° = -285.8$ kJ .... (ii)

$CO_2(g) + 2H_2O(1) \longrightarrow CH_4(g) + 2O_2(g)$; $\Delta_r H° = +890.3$ ..... (iii)

$C_{(graphite)} + 2H_2(g) \longrightarrow CH_4(g)$; $\Delta_r H° = ?$ .... (iv)

eq. (iv) = eq. (i) + 2 × eq (ii) + eq. (iii)

$= -393.5 + 2(-285.8) + 890.3$

$= -74.8$ kJ/mol

Ans. [4]

Sol. $\underset{1gm}{M_2CO_3} + HCl \longrightarrow \underset{.01186}{CO_2}$

POAC on carbon

$$\frac{1}{x} = \frac{.01186}{1} \Rightarrow x = 84.3$$

Ans. [1]

Sol. FLOT (According to first law of thermodynamics)

$\Delta E = q + w$

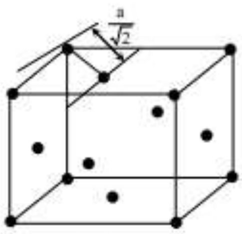
If $q = 0$, $\Delta E = w$ $\therefore$ adiabatic process

Ans. [4]

Sol. Facts

Ans. [2]

Sol. $\therefore$ nearest distance = $\frac{a}{\sqrt{2}}$



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Ans. [3]

Sol. Less is the SRP, more is the reducing power & strongest is the reducing agent.

Ans. [2]

Sol. $\Delta T_f = i \times K_f \times m$

$0.45 = \frac{i \times 5.12 \times 0.2 \times 1000}{60 \times 20}$

$i = .527$

$\beta = \frac{1-i}{1-1/n} = \frac{1-.527}{1-1/2} = .946$ or 94.6 %

Ans. [2]

Sol. $r = 0.529 \times \frac{n^2}{z}$ Å

$= 0.529 \times \frac{(2)^2}{1} = 2.116$ Å $\cong$ 2.12 Å

Ans. [2]

Sol.

R_1	R_2
A	A
E_a+10	E_a
k_1	k_2

$k_1 = A\,e^{-(Ea+10)/RT}$

$k_2 = A\,e^{-Ea/RT}$

$\frac{k_2}{k_1} = e^{(-Ea+Ea+10)/RT}$

$\frac{k_2}{k_1} = e^{10/RT} = e^{10\times10^3/8.314\times300}$

$= e^{10000/2494.2} = e^4$

$\boxed{\ln\frac{k_2}{k_1} = 4}$

Ans. [4]

Sol. $pH = \frac{1}{2}pK_w + \frac{1}{2}pK_a - \frac{1}{2}pK_b$

$= \frac{1}{2}\times14 + \frac{1}{2}\times3.2 - \frac{1}{2}\times3.4$

$= 7 + 1.6 - 1.7$

$= 6.9$

Ans. [3]

Sol. It is the best possible option but, it should be bonus because Li_2CO_3 is less basic and $MgCO_3$ is basic

Ans. [4]

Sol. CO is diamagnetic because all electrons are paired in it.

Ans. [3]

Sol. $\overset{+4}{Xe}F_4 + \overset{+1}{O_2}F_2 \rightarrow \overset{+6}{Xe}F_6 + \overset{0}{O_2}$

This reaction is a redox reaction

Ans. [1]

Sol. F^- ion concentration above 2 ppm causes brown mottling of teeth

Ans. [1]

Sol. $Cl_2 + NaOH_{(cold/dilute)} \rightarrow NaCl + NaOCl + H_2O$

Ans. [2]

Sol. $ZnO + Na_2O \rightarrow Na_2ZnO_2$

acid

$ZnO + CO_2 \rightarrow ZnCO_3$

Base

ZnO is an amphoteric oxide

Ans. [2]

Sol. $\underset{\text{(Oxalate-ion)}}{\underset{Na_2C_2O_4}{[X]}} \xrightarrow{\text{Conc. } H_2SO_4} Na_2CO_3 + H_2C_2O_4 \rightarrow \underset{\text{Effervesce}}{CO_2\uparrow}$

$Na_2C_2O_4 \xrightarrow{CaCl_2} \underset{\text{White ppt}}{CaC_2O_4\downarrow} + 2NaCl$

$\underset{\text{Pink Colour}}{KMnO_4} + \underset{\text{Oxalate ion}}{C_2O_4^{-2}} \xrightarrow{H^+} CO_2\uparrow + \underset{\text{Colourless}}{Mn^{+2}}$

Ans. [2]

Sol. $CoCl_3.6H_2O + AgNO_3 \text{ (excess)} \longrightarrow AgCl$

100 ml, 0.1 M — 1.2×10^{22} ion

No. of moles = 0.01 mol — $\dfrac{1.2\times10^{22}}{6\times10^{23}} = 0.02$ mol

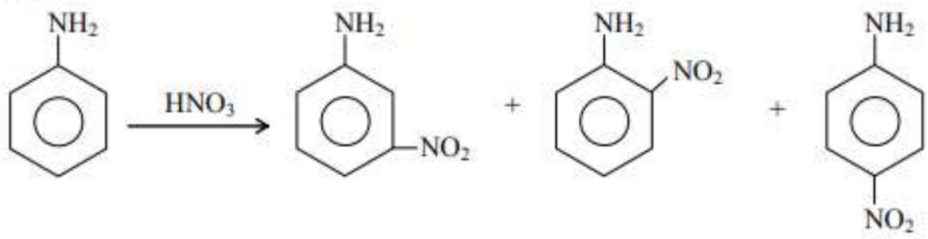
$\because$ 0.01 mol of $CoCl_3.6H_2O$ produce 0.02 mol of AgCl

$\therefore$ 1 mol of $CoCl_3.6H_2O$ produce 2 mol of AgCl

$\therefore$ Correct complex is $[Co(H_2O)_5Cl]Cl_2.H_2O$

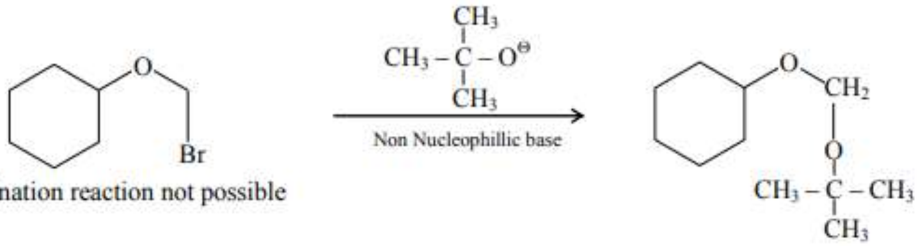
Ans. [1]

Sol.



[47%] [2%] [51%]

Ans. [3]

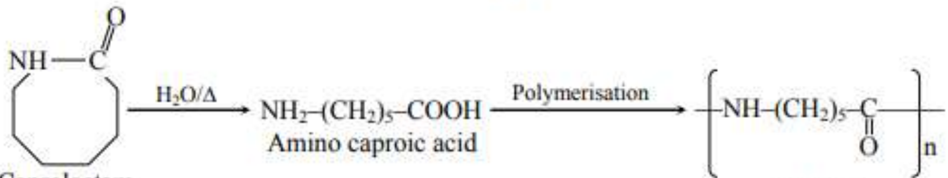


Sol. Elimination reaction not possible

It do not give unsaturation test

Ans. [3]

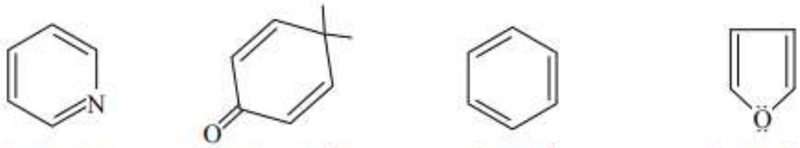
Sol. Nylon-6 is formed by monomer which is obtain by hydrolysis of CAPROLACTAM



Caprolactam Nylon-6

Ans. [2]

Sol.



Aromatic Non Aromatic Aromatic Aromatic

2 is least stable as other are aromatic and 2 is non aromatic

Ans. [4]

Sol. Rate of SN_1 reaction $\propto$ stability of carbocation

$CH_3CH(Cl)CH_2CH_3$ (I) $\longrightarrow CH_3-\overset{\oplus}{C}H-CH_2-CH_3+Cl^{\ominus}$

2° Carbocation

$CH_3-CH_2-CH_2-Cl \longrightarrow CH_3-CH_2-\overset{\oplus}{C}H_2$

1° Carbocation

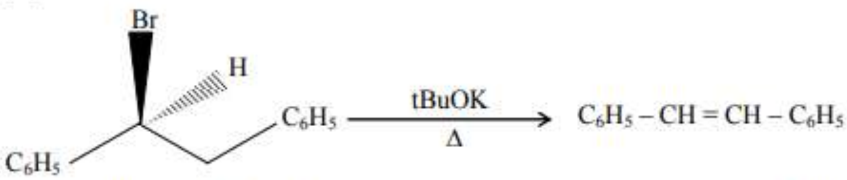
$CH_3O-C_6H_4-CH_2-Cl \longrightarrow CH_3O-C_6H_4-\overset{\oplus}{C}H_2$

Resonance Stabilize

Order of SN_1 = II < I < III

Ans. [4]

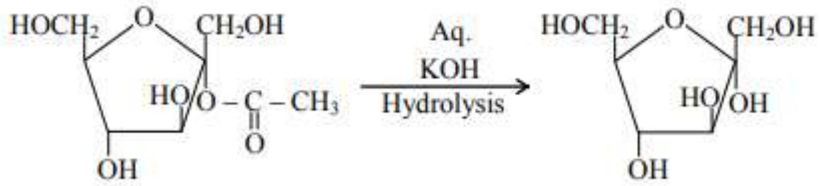
Sol.



It is example of E_2 elimination as t butoxide is stronger base and heating is also used.

Ans. [3]

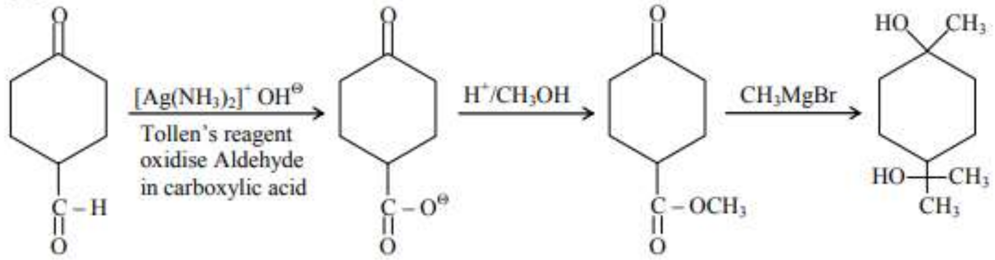
Sol.



It is hemiacetal that is why it can behave as reducing sugar.

Ans. [3]

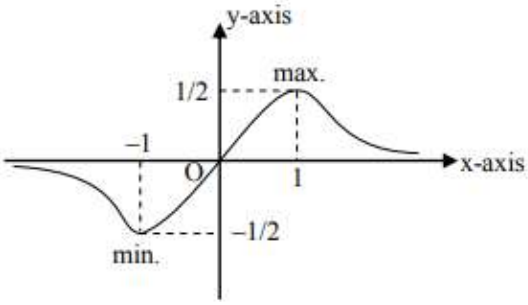
Sol.



MATHEMATICS

Ans. [2]

Sol.

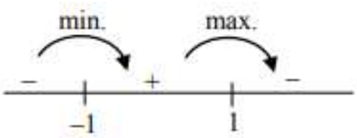


$f(x) = \frac{x}{1+x^2}$ (odd)

(symmetry about origin)

$$\frac{dy}{dx} = \frac{(1+x^2)\cdot 1 - x(0+2x)}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2} = 0 \Rightarrow x = 1, -1$$



Line parallel to x-axis cuts the graph more than one points hence function is many one.

Range = $\left[-\frac{1}{2}, \frac{1}{2}\right]$ = codomain hence function is onto

Sol. $x(x+1)+(x+1)(x+2)+ \ldots + (x+(n-1))(x+n) = 10n$

After simplify

$nx^2 + (1+3+5+7+\ldots\ldots+(2n-1))x + (0\cdot1 + 1\cdot2 + 2\cdot3 + \ldots + (n-1)n) = 10n$

$nx^2 + n^2x + \frac{n(n^2-1)}{3} - 10n = 0$

$x^2 + nx + \frac{n^2-1-30}{3} = 0$

$x^2 + nx + \frac{n^2-31}{3} = 0$

Put n = 11 (where $n \in I^+$)

$x^2 + 11x + \frac{121-31}{3} = 0$

$x^2 + 11x + 30 = 0$

$(x+6)(x+5) = 0$

i.e. x = –5, –6 (Two consecutive integral solutions)

So, n = 11

Ans. **[4]**

Sol. Apply operation $C_1 = C_1 + C_2 + C_3$

$$\begin{vmatrix} 3 & 1 & 1 \\ 0 & -(1+\omega^2) & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix} = 3k$$

$$\begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix} = 3k \text{ (Because } 1+\omega+\omega^2 = 0)$$

open by C_1

$3(\omega^2 - \omega^4) = 3k$

$3(\omega^2 - \omega) = 3k$

$3(-1-\omega-\omega) = 3k$

$-3(1+2\omega) = 3k$ Given that $2\omega + 1 = z$

$-3z = 3k$

$k = -z$

Ans. **[1]**

Sol. $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Ans. [3]

Sol. $\Delta = 0$ and at the one of Δ_1 or Δ_2 or $\Delta_3 \neq 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$1[a - b] - 1[1 - a] + 1[b - a^2] = 0$

$2a - b - 1 + b - a^2 = 0$

$a^2 - 2a + 1 = 0$

$a = 1$

$x + y + z = 1$

$x + y + z = 1$

$x + by + z = 0$

only one value of b, S is singleton set

Ans. [4]

Sol.

X	Y
7 friends	7 friends
3M 4L	4M 3L

Case I : 3L from X side and 3M from Y side

$^4C_3 \times {}^4C_3 = 4 \times 4 = 16$

Case II : 3M from X side and 3L from Y side

$^3C_3 \times {}^3C_3 = 1 \times 1 = 1$

Case III : 2L and 1M from X side and 2M and 1L from Y side

$(^4C_2 \times {}^3C_1) \times (^4C_2 \times {}^3C_1) = (6 \times 3) \times (6 \times 3) = 18 \times 18 = 324$

Case IV : 2M and 1L from X side and 1M and 2L from Y side

$(^3C_2 \times {}^4C_1) \times (^4C_1 \times {}^3C_2) = (3 \times 4) \times (4 \times 3) = 12 \times 12 = 144$

Total number of ways = Case I + Case II + Case III + Case IV

= 16 + 1 + 324 + 144

= 485

Ans. [3]

Sol. $(^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10}) - (^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{10})$

$= \frac{1}{2}\left[2 \times {}^{21}C_1 + 2 \times {}^{21}C_2 + ... + 2 \times {}^{21}C_{10}\right] - (2^{10} - 1)$

$= \frac{1}{2}\left[{}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10} + {}^{21}C_{11} + ... + {}^{21}C_{20} + {}^{21}C_{21} - ({}^{21}C_0 + {}^{21}C_{21})\right] - (2^{10} - 1)$

$= \frac{1}{2}(2^{21} - 2) - (2^{10} - 1)$

$= 2^{20} - 1 - 2^{10} + 1$

$= 2^{20} - 2^{10}$

Ans. [1]

Sol. $225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$450a^2 + 18b^2 + 50c^2 - 150ac - 10ab - 30bc = 0$

$(15a - 3b)^2 + (3b - 5c)^2 + (15a - 5c)^2 = 0$

$(15a - 3b)^2 = 0,\ (3b - 5c)^2 = 0,\ (15a - 5c)^2 = 0$

$15a = 3b,\ 3b = 5c$

$15a = 3b = 5c$

$\frac{a}{1} = \frac{b}{5} = \frac{c}{3} = k$ (let)

$a = k, b = 5k, c = 3k$

Then a, c and b are in A.P.

Ans. [4]

Sol. As $a + b + c = 3$

So, $f(1) = 3$

$f(x + y) = f(x) + f(y) + xy$

Put $x = 1, y = 1$

$f(2) = 2f(1) + 1 = 7$

Put $x = 2, y = 1$

$f(3) = f(2) + f(1) + 2 = 12$

Put $x = 2,\ y = 2$

$f(4) = 2f(2) + 4 = 18$

So, $\sum_{n=1}^{10} f(n) = f(1) + f(2) + f(3) + ... + f(10)$

Let $S = 3 + 7 + 12 + 18 + ... + f(n)$

$S = \quad 3 + 7 + 12 + ... + f(n)$

$0 = 3 + 4 + 5 + 6 + ... - f(n)$

$f(n) = 3 + 4 + 5 + 6 + ... = \frac{n}{2}[6 + (n - 1)]$

$$f(n) = \frac{n(n+5)}{2}$$

$$\sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n$$

$$= \frac{1}{2} \times \frac{10 \times 11 \times 21}{6} + \frac{5}{2} \times \frac{10 \times 11}{2}$$

$$= 330$$

Ans. **[1]**

Sol. Put $x = \frac{\pi}{2} + h$

$$\lim_{h\to 0}\frac{\cot\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}+h\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^3}$$

$$\lim_{h\to 0}\frac{-\tan h+\sin h}{-8h^3}$$

$$\lim_{h\to 0}\frac{\tan h-\sin h}{8h^3}$$

by expansion method

$$\lim_{h\to 0}\frac{\left(h+\frac{h^3}{3}+\frac{2}{15}h^5....\right)-\left(h-\frac{h^3}{3!}+\frac{h^5}{5!}...\right)}{8h^3}$$

$$\lim_{h\to 0}\frac{h^3\left(\frac{1}{3}+\frac{1}{3!}\right)+h^5\left(\frac{2}{15}-\frac{1}{5!}\right)+...}{8h^3}$$

$$=\frac{\frac{1}{3}+\frac{1}{6}+0}{8}=\frac{1}{16}$$

Ans. **[4]**

Sol. $y=\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$

$$=\tan^{-1}\left(\frac{2\cdot 3x\sqrt{x}}{1-(3x\sqrt{x})^2}\right)$$

$$=2\tan^{-1}(3x\sqrt{x})$$

$$\frac{dy}{dx}=\frac{2}{1+(3x\sqrt{x})^2}\cdot 3(x\cdot\frac{1}{2\sqrt{x}}+\sqrt{x}\cdot 1)\quad \text{(differentiating w.r.t. x)}$$

$$=\frac{6}{1+9x^3}\left(\frac{\sqrt{x}}{2}+\sqrt{x}\right)$$

$$=\frac{9\sqrt{x}}{1+9x^3}$$

$$=\sqrt{x}\cdot\frac{9}{1+9x^3}$$

Ans. [1]

Sol. $y(x-2)(x-3) = x+6$

at y axis, $x = 0$

$y(-2)(-3) = 0 + 6$

$y = 1$

Now $y(x^2 - 5x + 6) = x + 6$

$$y = \frac{x+6}{x^2-5x+6}$$

$$\frac{dy}{dx} = \frac{(x^2-5x+6)\cdot 1-(x+6)(2x-5)}{(x^2-5x+6)^2}$$

at $x = 0, y = 1$ $\quad = \frac{6-(6)(-5)}{6^2} = 1$

equation of normal

$y - 1 = -1(x - 0)$

$x + y = 1$

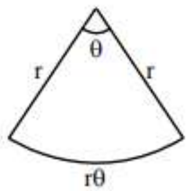
passes $\left(\frac{1}{2}, \frac{1}{2}\right)$ (by option)

Ans. [2]

Sol. Given

$r + r + r\theta = 20$

$$\theta = \frac{20-2r}{r}$$



Area $= \frac{1}{2}r^2\theta$

$$= \frac{1}{2}r^2\cdot\left(\frac{20-2r}{r}\right)$$

$$z = \frac{1}{2}(20r - 2r^2)$$

$$\frac{dz}{dr} = \frac{1}{2}(20-4r) = 0 \Rightarrow r = 5$$

at $r = 5$, $\theta = 2$, $\frac{d^2z}{dr^2} < 0$ (hence maxima)

maximum area

$$z = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}\times 5^2 \times 2 = 25m^2$$

Ans. [1]

Sol.
$$I_n = \int \tan^n x \, dx$$

$$I_4 + I_6 = \int (\tan^4 x + \tan^6 x) dx$$

$$= \int \tan^4 x (1 + \tan^2 x) dx$$

$$= \int \tan^4 x \cdot \sec^2 x \, dx \qquad \text{put } t = \tan x$$

$$= \int t^4 \cdot dt$$

$$= \frac{t^5}{5} + C$$

$$= \frac{\tan^5 x}{5} + C$$

On comparison, we get

$$a = \frac{1}{5}, \quad b = 0$$

Ans. [1]

Sol.
$$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$$

$$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos(\pi - x)} \qquad \text{by using } \int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$2I = \int_{\pi/4}^{3\pi/4} \left(\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} \right) dx = \int_{\pi/4}^{3\pi/4} \left(\frac{2}{1 - \cos^2 x} \right) dx$$

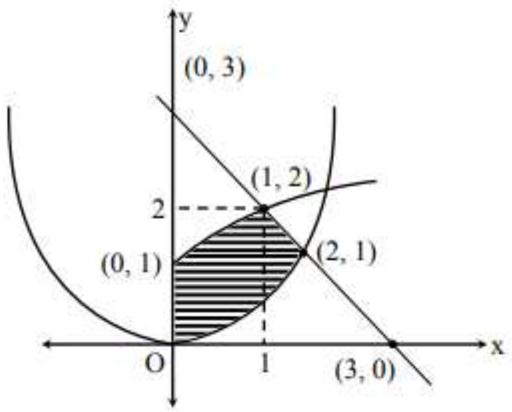
$$2I = 2 \int_{\pi/4}^{3\pi/4} \operatorname{cosec}^2 x \, dx$$

$$I = -(\cot x)_{\pi/4}^{3\pi/4}$$

$$-(-1 - 1) = 2$$

Ans. [3]

Sol.



Required Area = $\int_0^1\left(1+\sqrt{x}-\frac{x^2}{4}\right)dx+\int_1^2\left(3-x-\frac{x^2}{4}\right)dx$

$=\left[x+\frac{x^{3/2}}{3/2}-\frac{x^3}{12}\right]_0^1+\left[3x-\frac{x^2}{2}-\frac{x^3}{12}\right]_1^2$

$=\frac{19}{12}+\frac{11}{12}=\frac{5}{2}$

Sol. $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$

$\frac{dy}{dx}=\frac{-(y+1)\cos x}{2+\sin x}$

$\Rightarrow \int\frac{dy}{y+1}=-\int\left(\frac{\cos x}{2+\sin x}\right)dx$

$\Rightarrow \log(y+1)=-\log(2+\sin x)+\log c$

$\Rightarrow y+1=\frac{c}{2+\sin x}$ ...(1)

Given that $y(0)=1$

$\therefore\ 1+1=\frac{c}{2} \Rightarrow c=4$

$\therefore$ Equation of curve

$y+1=\frac{4}{2+\sin x}$

at $x=\frac{\pi}{2}$ $\Rightarrow y+1=\frac{4}{2+1}$

$\Rightarrow y=\frac{4}{3}-1$

$y=\frac{1}{3}$

Ans. [3]

Sol. Area

$$\frac{1}{2}\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 28$$

$k(k-2) + 3k(5+k) + 1(10+k^2) = \pm 56$

$k^2 - 2k + 15k + 3k^2 + 10 + k^2 = \pm 56$

$5k^2 + 13k + 10 = \pm 56$

$5k^2 + 13k + 66 = 0$

$D = 169 - 4 \times 5 \times 66 < 0$

No solution

$5k^2 + 13k - 46 = 0$

$5k^2 + 23k - 10k - 46 = 0$

$k(5k + 23) - 2(5k + 23) = 0$

$(5k + 23)(k - 2) = 0$

$k = 2$ (k is integer)

Hence co-ordinate

(2, –6) (5, 2) (–2, 2)

$E(2, \beta)$

$AD \perp BC$

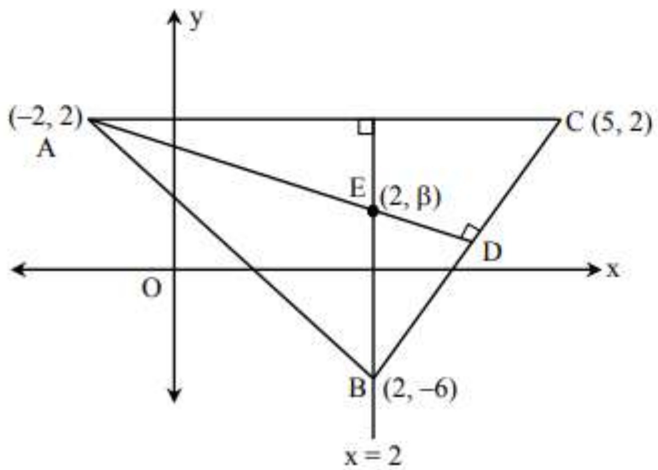
$$\frac{\beta - 2}{2+2} \times \frac{8}{3} = -1$$

$$\frac{\beta - 2}{4} = \frac{-3}{8}$$

$$\beta - 2 = -\frac{3}{2}$$

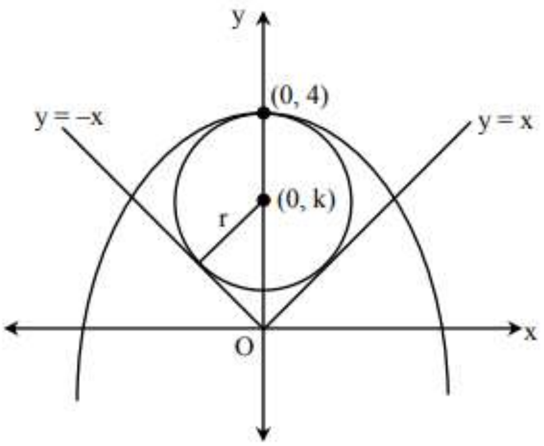
$$\beta = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\left(2, \frac{1}{2}\right)$$



Ans. **[2]**

Sol.



Circle touches the line

By graph radius = 4 – k

Perpendicular distance from centre = radius

$\Rightarrow \; 4-k=\left|\dfrac{0-k}{\sqrt{2}}\right|$

$\Rightarrow \; 16+k^2-8k=\dfrac{k^2}{2}$

$\Rightarrow \; k^2-16k+32=0$

$\Rightarrow \; k=\dfrac{16\pm\sqrt{256-4(32)}}{2}$

$\Rightarrow \; k=\dfrac{16\pm\sqrt{128}}{2}$

$\Rightarrow \; k=\dfrac{16\pm8\sqrt{2}}{2}$

$\Rightarrow \; k=8\pm4\sqrt{2}$

$\Rightarrow \; k=8-4\sqrt{2}$ (k should be $0<k<4$)

Radius $=4-k$

$=4-(8-4\sqrt{2})$

$=4(\sqrt{2}-1)$

Ans. **[1]**

Sol. Let the equation of ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

given that $e=\dfrac{1}{2}$

and directrix $\left\{\begin{matrix} x=-a/e \\ \text{or} \\ x=-4 \end{matrix}\right\}$

$\Rightarrow \frac{a}{e} = 4$

$\Rightarrow a = 2$

Now, $b^2 = a^2 (1 - e^2)$

$$b^2 = 4\left(1 - \frac{1}{4}\right) = 3$$

Equation of ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

diff. w.r.t. x

$$\frac{2x}{4} + \frac{2y}{3}\frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_{(1,\,3/2)} = -\frac{1}{2}$$

$\therefore$ Equation of normal at $\left(1, \frac{3}{2}\right)$ is

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)}(x - x_1)$$

$\Rightarrow \quad y - \frac{3}{2} = 2(x - 1)$

$\Rightarrow \quad 2y - 3 = 4x - 4$

$\Rightarrow \quad 4x - 2y = 1$

Ans. [1]

Sol. Let the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

given that foci $(\pm ae, 0) = (\pm 2, 0)$

$\Rightarrow \quad ae = 2$

Hyperbola passes through $P(\sqrt{2}, \sqrt{3})$

$\therefore \frac{2}{a^2} - \frac{3}{b^2} = 1$

$\Rightarrow \frac{2}{a^2} - \frac{3}{a^2(e^2-1)} = 1$

$\Rightarrow \frac{2}{a^2} - \frac{3}{4-a^2} = 1$

$\Rightarrow 8 - 2a^2 - 3a^2 = 4a^2 - a^4$

$\Rightarrow a^4 - 9a^2 + 8 = 0$

$\Rightarrow (a^2 - 8)(a^2 - 1) = 0$

$\Rightarrow a^2 = 8$ and $a^2 = 1$

$a^2 = 8$	$a^2 = 1$
$b^2 = a^2(e^2 - 1)$	$b^2 = a^2(e^2 - 1)$
$b^2 = 4 - 8$	$b^2 = 4 - 1 = 3$
$b^2 = -4$ (not possible)	$\therefore$ Equation of hyperbola
	$\frac{x^2}{1} - \frac{y^2}{3} = 1$
	tangent at $P(\sqrt{2}, \sqrt{3})$
	$T = 0$
	$\sqrt{2}x - \frac{y}{\sqrt{3}} = 1$

By option it passes through $(2\sqrt{2}, 3\sqrt{3})$

Ans. [1]

Sol. $\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} - \hat{j}(-7) + 3\hat{k}$

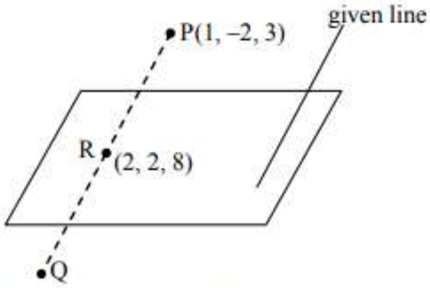
Equation of plane $5(x-1) + 7(y+1) + 3(z+1) = 0$

$$5x + 7y + 3z + 5 = 0$$

Perpendicular distance of the plane from $(1, 3, -7)$ is $= \frac{|5+21-21+5|}{\sqrt{25+49+9}} = \frac{10}{\sqrt{83}}$

Ans. [1]

Sol.



The equation of line PR

$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = k$

let R(k + 1, 4k – 2, 5k + 3)

it lies on the plane 2x + 3y – 4z + 22 = 0

$\therefore\ 2(k+1) + 3(4k-2) - 4(5k+3) + 22 = 0$

$\Rightarrow -6k + 6 = 0$

$\Rightarrow k = 1$

$\therefore$ R(2, 2, 8)

Image of P in the plane is (R is the mid-point of PQ)

$\therefore$ Q(3, 6, 13)

$PQ = \sqrt{4+64+100} = \sqrt{168} = 2\sqrt{42}$

Ans. **[1]**

Sol. $|\vec{a} \times \vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$|\vec{a} \times \vec{b}| = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

given $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$

$|\vec{a} \times \vec{b}|\,|\vec{c}| \sin 30^\circ = 3$

$\Rightarrow \quad 3|\vec{c}| \cdot \frac{1}{2} = 3$

$\Rightarrow \quad |\vec{c}| = 2$

Now $\quad |\vec{c} - \vec{a}| = 3$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\,\vec{a}\,.\,\vec{c} = 9$

$\Rightarrow \quad 4 + 9 - 2\,\vec{a}\,.\,\vec{c} = 9$

$\Rightarrow \quad \vec{a}\,.\,\vec{c} = 2$

Ans. **[4]**

Sol. Total no. of balls = 25

(15 green and 10 yellow balls)

(Varience) $\sigma^2 = npq$

where $n \rightarrow$ No. of Trial

$p \rightarrow$ Probability of happening of that event

$q \rightarrow$ Probability of not happening of that event

$n = 10,\ p = \frac{15}{25} = \frac{3}{5},\ q = \frac{10}{25} = \frac{2}{5}$

So, $\quad \sigma^2 = 10 \times \frac{3}{5} \times \frac{2}{5}$

$\sigma^2 = \frac{60}{25} = \frac{12}{5}$

Ans. **[1]**

Sol. P(Exactly one of A or B occurs) = $P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ ... (1)

P(Exactly one of B or C occurs) = $P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ ... (2)

P(Exactly one of C or A occurs) = $P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ ... (3)

Adding (1), (2) and (3)

$2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \frac{3}{4}$

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$

P(All the three events occurs simultaneously) = $P(A \cap B \cap C) = \frac{1}{16}$

P(Atleast one of the events occurs) = $P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

Ans. **[4]**

Sol. $n(S) = {}^{11}C_2 = 55$

Favorable events $\begin{cases} (0, 4) \\ (0, 8) \\ (2, 6), (2, 10) \\ (4, 8), (6, 10) \end{cases}$

So, required probability = $\frac{\text{Fav. Events}}{\text{Total Events}} = \frac{6}{55}$

Ans. **[3]**

Sol. $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$\Rightarrow 5\left[\frac{\sin^2 x}{\cos^2 x} - \cos^2 x\right] = 2(2\cos^2 x - 1) + 9$

$\Rightarrow 5[(1 - \cos^2 x) - \cos^4 x] = 4\cos^4 x - 2\cos^2 x + 9\cos^2 x$

$\Rightarrow 9\cos^4 x + 12\cos^2 x - 5 = 0$

$\Rightarrow 9\cos^4 x + 15\cos^2 x - 3\cos^2 x - 5 = 0$

$\Rightarrow 3\cos^2 x\,(3\cos^2 x + 5) - (3\cos^2 x + 5) = 0$

$\Rightarrow \cos^2 x = \frac{1}{3}$

$\Rightarrow \cos 2x = 2\cos^2 x - 1$

$\Rightarrow \cos 2x = \frac{2}{3} - 1 = \frac{-1}{3}$

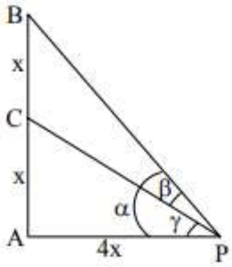
Now $\cos 4x = 2\cos^2 2x - 1$

$\cos 4x = 2\left(\frac{1}{9}\right) - 1$

$\cos 4x = \frac{-7}{9}$

Ans. [2]

Sol.



$\beta = \alpha - \gamma$

$$\tan\beta = \frac{\tan\alpha - \tan\gamma}{1 + \tan\alpha . \tan\gamma} = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{8}} = \frac{2}{9}$$

Ans. [4]

Sol.

q	p	~p	$(p \to q)$	$(\sim p \to q)$	$(\sim p \to q) \to q$	$(p \to q) \to [(\sim p \to q) \to q]$
T	T	F	T	T	T	T
F	T	F	F	T	F	T
T	F	T	T	T	T	T
F	F	T	T	F	T	T

It is a tautology.