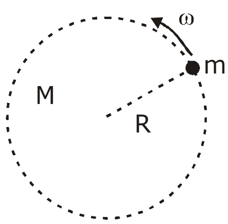
QUESTION PAPER WITH SOLUTION

PHYSICS - 2 Sep 2020. - SHIFT - 1

Q.1 The mass density of a spherical galaxy K varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as:

(1) $T^2 \propto R$ (2) $T^2 \propto R^3$ (3) $T^2 \propto \frac{1}{R^3}$ (4) $T \propto R$

Sol. **(1)**



Mass of galaxy $= \int_0^R \rho dv$

$= \int_0^R \frac{k}{r} 4\pi r^2 dr$

$= 4\pi k \int_0^R r dr$

$M = \frac{4\pi k R^2}{2} = k_1 R^2$

$F = m\omega^2 R$

$\frac{GMm}{R^2} = m\omega^2 R$

$\frac{Gk_1 R^2}{R^2} = \omega^2 R$

$\therefore \omega^2 = \frac{k_2}{R}$

$\omega = \sqrt{\frac{k_2}{R}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{k_2}}$

$T = k_3\sqrt{R}$

$T^2 \propto R$

Q.2 An amplitude modulated wave is represented by the expression v_m = 5(1 + 0.6 cos 6280t) sin (211 x 10^4 t) volts. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively :

(1) $\frac{3}{2}$V, 5V (2) 5V, 8V (3) 3V, 5V (4) $\frac{5}{2}$V, 8V

Sol. **(4)**

$$\frac{A_m}{A_c} = 0.6$$

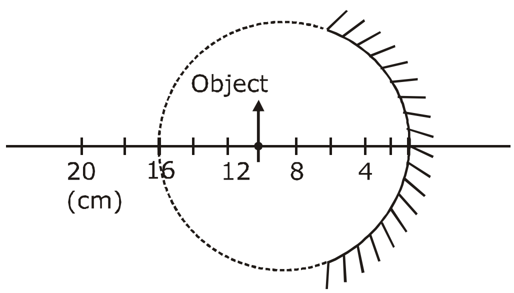
= (5+3 cos 6280t) sin (211×10^4 t)

maximum Amp. = 5+3 = 8 V

minimum Amp. = 5–3 = 2 V

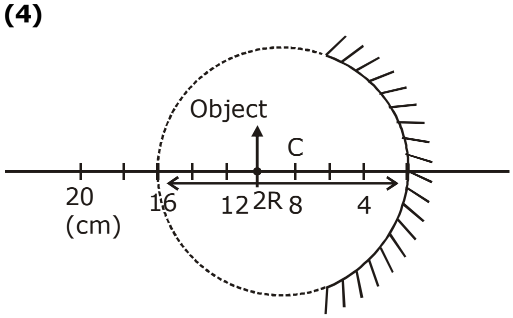
from the given option nearest value of minimum Amplitude = $\frac{5}{2}$V

Q.3 A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the minor, what will be the nature and magnification of the image of the object ? (Figure drawn as schematic and not to scale)



(1) Erect, virtual and unmagnified
(2) Inverted, real and magnified
(3) Erect, virtual and magnified
(4) Inverted, real and unmagnified

Sol. **(4)**

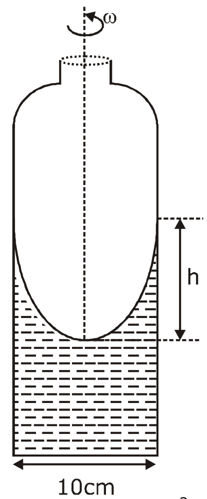


∴ beyond C i.e. $-\infty < u < C$

∴ real, inverted

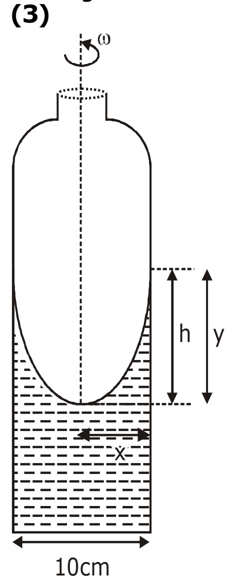
and unmagnified

Q.4 A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm an and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h (in cm) of liquid at the centre of vessel and at the side will be :



(1) $\frac{5\omega^2}{2g}$ (2) $\frac{2\omega^2}{25g}$ (3) $\frac{25\omega^2}{2g}$ (4) $\frac{2\omega^2}{5g}$

Sol. **(3)**



$y = \frac{\omega^2 x^2}{2g}$

at x = 5cm, y=h

$h = \frac{\omega^2 (5)^2}{2g} = \frac{25\omega^2}{2g}$

Q.5 If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be

(1) FA^2V^{-3} (2) FA^2V^{-2} (3) $FA^{-1}V^0$ (4) FA^2V^{-1}

Sol. **(3)**

$Y = k\,[F]^x\,[A]^y\,[V]^z$

$[ML^1T^{-2}] = [MLT^{-2}]^x\,[L^2]^y\,[LT^{-1}]^z$

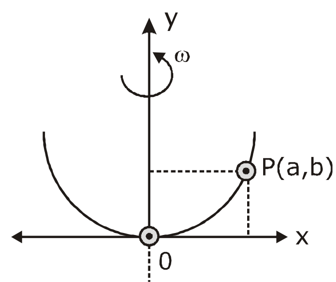
$[ML^1T^{-2}] = [M^x\,L^{x+2y+z}\,T^{-2x-z}]$

$x = 1,\ -2x-z = -2,\ x + 2y + z = -1$

$\Rightarrow z = 0$

$\Rightarrow y = -1$

Q.6 A bead of mass m stays at point P (a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction):



(1) $\sqrt{\frac{2g}{C}}$ (2) $2\sqrt{gC}$ (3) $\sqrt{\frac{2gC}{ab}}$ (4) $2\sqrt{2gC}$

Sol. **(4)**

$y = 4\,cx^2$

$\frac{dy}{dx} = 8cx$

$\tan\theta = \frac{m\omega^2 a}{mg}$

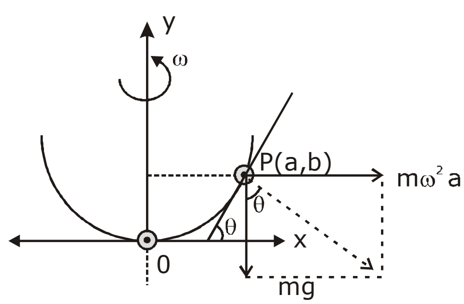
$\tan\theta = \frac{dy}{dx} = 8cx$

$8\,cx = \frac{\omega^2 a}{g}$

$(x = a),\ 8\,c\,a = \frac{\omega^2 a}{g}$

$\sqrt{8cg} = \omega$

$2\sqrt{2gc} = \omega$



Q.7 Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ?

(1) P : Small retentivity, large coercivity (2) P : Large retentivity, large coercivity

(3) T : Large retentivity, large coercivity (4) T : Large retentivity, small coercivity

Sol. **(2)**

Permanent magnet must retain for long use and should not be easily demagnetised.

Q. 8 Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to :

(1) 2.05 μm (2) 2.87 nm (3) 2 nm (4) 1.27 μm

Sol. **(4)**

given, d = 1mm

λ = 632.8 nm

D = 100cm

y = 1.27 mm

$\Delta x = d\sin\theta$

$\because (\theta = \text{small})$

$\Delta x = d\tan\theta$

$$\Delta x = \frac{dy}{D} = \frac{1x10^{-3} \times 1.27 \times 10^{-3}}{100 \times 10^{-2}}$$

$= 1.27 \times 10^{-6}$ m

= 1.27 μm

Q.9 A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is:

(1) 11 (2) 13 (3) 15 (4) 20

Sol. **(3)**

$$U = n_1C_{v_1}T + n_2C_{v_2}T$$

$$= 3 \times \frac{5}{2}RT + 5x\frac{3}{2}RT$$

$$= \frac{30}{2}RT = 15RT$$

Q. 10 A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is 1.02×10^{-8} J/m^3 in vacuum. The amplitude of the magnetic field of the wave is close

$(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$ and speed of light $= 3 \times 10^8$ ms^{-1}):

(1) 160 nT (2) 150 nT (3) 180 nT (4) 190 nT

Sol. (1)

energy density $= \frac{B_0^2}{2\mu_0}$ ...(1)

& $C = \frac{1}{\sqrt{\mu_0 \in_0}}$ ....(2)

$\mu_0 = \frac{1}{C^2 \in_0}$

$B = \sqrt{U \times 2\mu_0}$

$= \sqrt{1.02 \times 10^{-8} \times 2 \times \frac{1}{9 \times 10^{16}} 4\pi \times 9 \times 10^9}$

$= \sqrt{25.62 \times 10^{-15}}$

$\cong \sqrt{25600 \times 10^{-18}}$

$\cong 160 \times 10^{-9}$

= 160nT

Q. 11 Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity ρ_C, ρ_T, ρ_m and ρ_A respectively. Then :

(1) $\rho_C > \rho_A > \rho_T$ (2) $\rho_A > \rho_M > \rho_C$ (3) $\rho_A > \rho_T > \rho_C$ (4) $\rho_M > \rho_A > \rho_C$

Sol. (4)

(Theoretical concept)

Q.12 A beam of protons with speed 4 x 10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to : (Mass of the pr oton $=1.67 \times 10^{-27}$ kg, charge of the proton $=1.69 \times 10^{-19}$ C)

(1) 4 cm (2) 2 cm (3) 12 cm (4) 5 cm

Sol. **(1)**

pitch = Vcos 60º , $T = \frac{V}{2}\frac{2\pi m}{eB}$

$= 4 \times 10^5 \times \frac{1}{2} \times \frac{2\pi}{0.3}\left(\frac{m}{e}\right)$

$= \frac{4\pi \times 10^5 \times 10^{-8}}{0.3}$

$= \frac{4 \times 3.14 \times 10^{-3}}{3 \times 10^{-1}}$

$\simeq 4 \times 10^{-2}$ m

$\approx$ 4cm

Q.13 Two identical strings X and Z made of same material have tension T_x and T_z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_x/T_z is :

(1) 2.25 (2) 1.25 (3) 0.44 (4) 1.5

Sol. **(1)**

$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

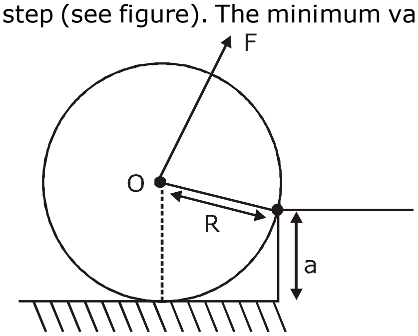
given, $\mu_x = \mu_z$ & $L_x = L_z$ as identical

$\therefore f \propto \sqrt{T}$

$\Rightarrow \frac{T_x}{T_z} = \frac{f_x^2}{f_z^2} = \left(\frac{450}{300}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

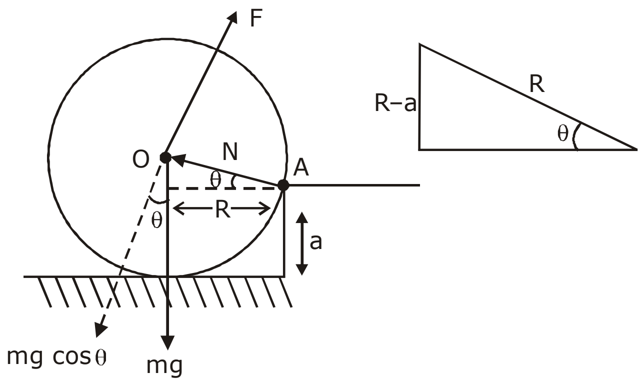
$\frac{T_x}{T_y}$ = 2.25

Q.14 A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is



(1) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2 - 1}$ (2) $Mg\sqrt{1 - \frac{a^2}{R^2}}$ (3) $Mg\frac{a}{R}$ (4) $Mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$

Sol. (4)



$$\cos\theta = \frac{\sqrt{R^2 - (R-a)^2}}{R}$$

$$= \sqrt{\frac{R^2}{R^2} - \left(\frac{R-a}{R}\right)^2}$$

$$= \sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

to pull up, $\tau_F \geq \tau_{mg}$

$FR \geq mg\cos\theta R$

for min F, $F_{min} = mg\cos\theta$

$$F_{min} = mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

Q.15 In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, $N = 6.023 \times 10^{26}$ per kilo mole and $1\ eV = 1.6 \times 10^{-19}$ J. The power output of the reactor is close to

(1) 60 MW (2) 54 MW (3) 125 MW (4) 35 MW

Sol. **(1)**

$$n(\text{moles}) = \frac{2kg}{235gm} = \frac{2000}{235}$$

no. of nucleus = $N_A \times n$

$$= 6.022 \times 10^{23} \times \frac{2000}{235}$$

$= 51.25 \times 10^{23}$

total energy released = $200 \times 51.25 \times 10^{23}$ MeV

$= 102.5 \times 10^{25}$ MeV

$= 102.5 \times 10^{25} \times 10^{6} \times 1.6 \times 10^{-16}$J

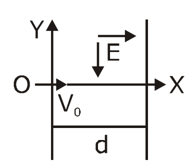
$= 164 \times 10^{6}$ MJ

$$\text{power} = \frac{164 \times 10^{6} MJ}{30 \times 24 \times 60 \times 60\, S}$$

$= 0.063 \times 10^{3}$ MW

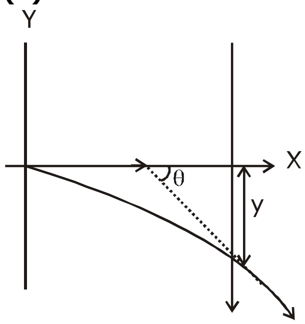
$\cong$ 60MW

Q.16 A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is :



(1) $y = \frac{qEd^2}{mV_0^2}x$ (2) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$ (3) $y = \frac{qEd}{mV_0^2}(x - d)$ (4) $y = \frac{qEd}{mV_0^2}x$

Sol. **(2)**



$-\ y = \frac{1}{2}at^2$

$-y = \frac{1}{2}\frac{qE}{m}t^2$ ...(1)

$X = V_o\ t$

$\Rightarrow t = \frac{x}{V_0}$ ...(2)

for $x \le d$,

$y = -\frac{1}{2}\frac{qE}{m}\frac{x^2}{V_0^2}$ ...(3)

$\left.\frac{dy}{dx}\right|_{x=d} = -\frac{1}{2}\frac{qE}{m} \times \left.\frac{2x}{V_0^2}\right|_{x=d}$

$\text{Slope} = m = \tan\theta = -\frac{qEd}{mV_0^2}$

equation of straight line,

$y = (\tan\theta)\ x + c$ ...(4)

$= -\left(\frac{qEd}{mv_0^2}\right)x + c$

(now for C, at x = d, $y = -\frac{qEd^2}{2mv_0^2}$ put in (4)

$-\frac{qEd^2}{2mv_0^2} = -\frac{qEd^2}{mV_0^2} + c$

$\Rightarrow c = \dfrac{qEd^2}{2mv_0^2}$

for x > d, as no $\vec{E}$

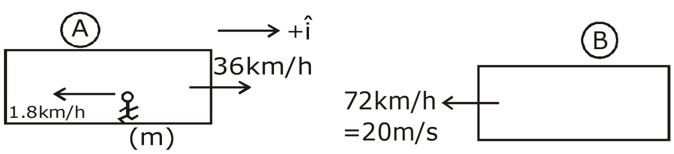
$$y = -\left(\frac{qEd}{mv_0^2}\right)x + \frac{qEd^2}{2mv_0^2}$$

$$y = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)$$

Q.17 Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(1) 29.5 ms^{-1} (2) 30.5 ms^{-1} (3) 31.5 ms^{-1} (4) 28.5 ms^{-1}

Sol. **(1)**



$$\overrightarrow{V_m} = \overrightarrow{V_{m/A}} + \overrightarrow{V_A}$$

$$= (-1.8\,\hat{i} + 36\hat{i})\text{km/ h}$$

$$= \left(-1.8 \times \frac{5}{18} + 36 \times \frac{5}{18}\right)\text{m / s}$$

$$= \left(-0.5\hat{i} + 10\hat{i}\right)\text{m/s}$$

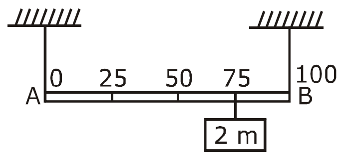
$$= 9.5\hat{i}\ \text{m/s}$$

$$\overrightarrow{V_{m/B}} = \overrightarrow{V_M} - \overrightarrow{V_B}$$

$$= 9.5\hat{i} - (-20\,\hat{i})\text{m/s}$$

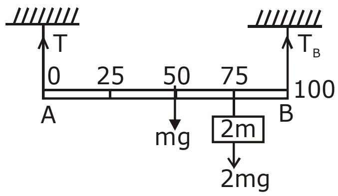
$$= 29.5\ \text{m/s}\ \hat{i}$$

Q.18 Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass `m` and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is:



(1) 0.75 mg (2) 0.5 mg (3) 1 mg (4) 2 mg

Sol. **(3)**



τ_{net} about B = 0

T × 100cm = mg × 50cm + 2mg × 25cm

T = 1 mg

Q.19 The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is: (VSD is vernier scale division)

(1) 3.21 cm (2) 3.07 cm (3) 2.99 cm (4) 3.2 cm

Sol. **(2)**

L.C. = 1MSD – 1VSD

L.C. = 0.1MSD

1 MSD = 1mm

L. C. = 0.1mm

+ve zero error = +7 × L.C.

= 0.7mm

Reading = (3.1cm + 4 × L.C) – zero error

= 3.1cm + 0.4mm – 0.7mm

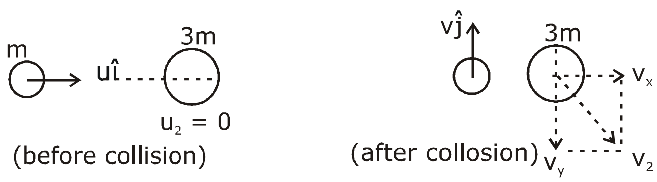
= 3.1cm – 0.03cm (as given 1 MSD = 1mm)

= 3.07 cm

Q. 20 A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3m at rest. It moves with a velocity $v\hat{j}$ after collision, then v is given by:

(1) $v = \frac{1}{\sqrt{6}}u$ (2) $v = \sqrt{\frac{2}{3}}u$ (3) $v = \frac{u}{\sqrt{3}}$ (4) $v = \frac{u}{\sqrt{2}}$

Sol. (4)



$\vec{p_i} = \vec{p_f}$

$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{V_2}$

$\frac{mu\hat{i}}{3m} - \frac{mv\hat{j}}{3m} = \vec{V_2}$

$\vec{V_2} = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$

now, K·E conserved as elastic collision

$\Sigma KE_i = \Sigma KE_f$

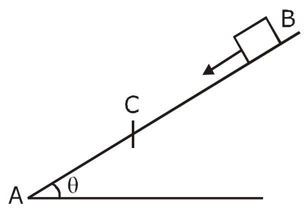
$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}3m\left(\frac{u^2}{9} + \frac{v^2}{9}\right)$

$\Rightarrow u^2 = v^2 + \frac{u^2}{3} + \frac{v^2}{3}$

$\frac{2}{3}u^2 = \frac{4}{3}v^2$

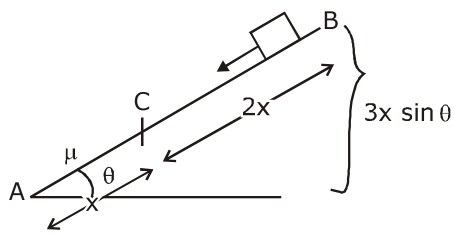
$\Rightarrow v = \frac{u}{\sqrt{2}}$

Q.21



A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by $\mu = k \tan\theta$. The value of k is __________.

Sol. **(3)**



from work energy theorem

$W_g + W_f = \Delta kE$

$mg\, 3x \sin\theta - \mu mg \cos\theta\, x = 0 - 0$

$\Rightarrow mg 3x \sin\theta = \mu mg \cos\theta x$

$3 \tan\theta = \mu$

$k = 3$

Q.22 An engine takes in 5 moles of air at 20°C and 1atm, and compresses it adiabaticaly to $1/10^{th}$ of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is ___________.

Sol. **(46)**

$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$= 293\left(\frac{V}{V/10}\right)^{\frac{7}{5}-1}$

$T_2 = 293 \times (10)^{2/5}$

$\Delta U = nC_v \Delta T = 5 \times \frac{5}{2} R \left(293 \times 10^{\frac{2}{5}} - 293\right)$

$$= \frac{25}{2} R \times 293 \left(10^{\frac{2}{5}} - 1\right) = \frac{25R}{2} \times 293(2.5 - 1)$$

$$= \frac{25 \times 8.314 \times 293 \times 1.5}{2}$$

= 45675 J = 46kJ

Q.23 When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V.

When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $\frac{V}{4}$.

If the threshold wavelength for the metallic surface is $n\lambda$ then value of n will be ____________.

Sol. **(9)**

$$\frac{hc}{\lambda} = \phi + eV \qquad \ldots(1)$$

$$\frac{hc}{3\lambda} = \phi + \frac{eV}{4} \qquad \ldots(2)$$

$$\frac{\text{eq.}(1)}{\text{eq.}(2)} \qquad 3 = \frac{\phi + eV}{\phi + \frac{eV}{4}}$$

$$3\phi + \frac{3eV}{4} = \phi + eV$$

$$2\phi = \frac{eV}{4}$$

$$\phi = \frac{eV}{8}$$

$$\frac{hc}{\lambda} = \frac{eV}{8} + eV$$

$$= \frac{9}{8} eV$$

$$\therefore eV = \frac{8}{9}\frac{hc}{\lambda}$$

so $\phi = \frac{hc}{\lambda} - \frac{8}{9}\frac{hc}{\lambda}$

$$\phi = \frac{1}{9}\frac{hc}{\lambda}$$

$$\frac{hc}{\lambda_{th}} = \frac{hc}{9\lambda}$$

$$\therefore \lambda_{th} = 9\lambda$$

Q.24 A circular coil of radius 10 cm is placed in uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2s. The maximum value of EMF induced (in μV) in the coil will be close to the integer ___________.

Sol. **(15)**

$\phi = BA \cos \omega t$

$$E = \frac{-d\phi}{dt} = BA\omega \sin \omega t$$

$$E_{max} = BA\,\omega \qquad \left(\omega = \frac{\pi}{0.2}\right)$$

$$= 3 \times 10^{-5} \times \pi R^2 \times \frac{\pi}{0.2}$$

$= 15 \times 10^{-6}$ V

$= 15\mu$V

Q.25 A 5μF capacitor is charged fully by a 220V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J then value of X to the nearest integer is ________.

Sol. **(40) Our Answer**

NTA Answer 36

heat = $U_i - U_f$

$$= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2}\frac{5 \times 2.5}{7.5}(220 - 0)^2$$

$$= \frac{5}{6} \times 220 \times 220 \times 10^{-6}\,J$$

$= 40{,}333.33 \times 10^{-6}$ J

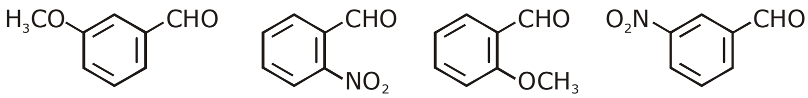
$$\frac{x}{100} = 0.4J$$

x = 40

QUESTION PAPER WITH SOLUTION

CHEMISTRY - 2 Sep 2020. - SHIFT - 1

1. The increasing order of the following compounds towards HCN addition is:

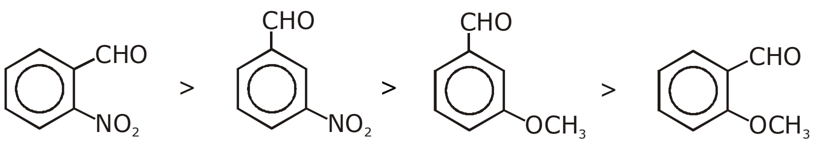


(i) (ii) (iii) (iv)

(1) (iii) < (i) < (iv) < (ii)
(2) (iii) < (iv) < (i) < (ii)
(3) (i) < (iii) < (iv) < (ii)
(4) (iii) < (iv) < (ii) < (i)

Sol. **1**

In HCN, CN^- is acts as nucleophile, attack first that –CHO group which has maximum positive charge. The magnitude of the (+ve) charge increases by –M and –I group. So reactivity order will be



(–M) (–I) (–I) (+M)

So, option (1) is correct answer.

2. Which of the following is used for the preparation of colloids?

(1) Van Arkel Method
(2) Ostwald Process
(3) Mond Process
(4) Bredig's Arc Method

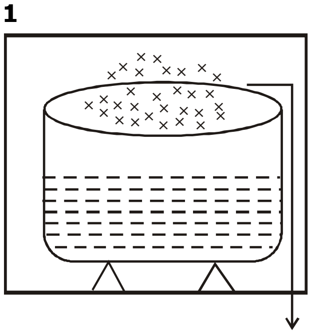
Sol. **4**

Bredig's Arc method
Chapter name surface chemistry

3. An open beaker of water in equilibrium with water vapour is in a sealed container. When a few grams of glucose are added to the beaker of water, the rate at which water molecules:

(1) leaves the vapour increases
(2) leaves the solution increases
(3) leaves the vapour decreases
(4) leaves the solution decreases

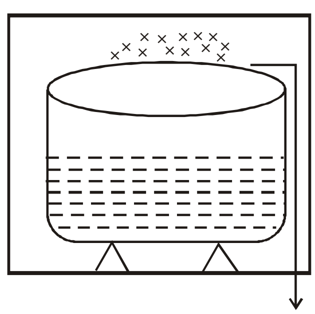
Sol. **1**



Vap. press = P^o

$H_2O(l) \rightleftharpoons H_2O(g)$

$K_p = P^o$



Vap. press = P_s

$H_2O(\ell) \rightleftharpoons H_2O(g)$

$K_P = P_s$

Backward shift

vapours ↓

Hence Rate at which water molecules leaves the vap. increases.

4. For octahedral Mn(II) and tetrahedral Ni(II) complexes, consider the following statements:

(I) both the complexes can be high spin.

(II) Ni(II) complex can very rarely be low spin.

(III) with strong field ligands, Mn(II) complexes can be low spin.

(IV) aqueous solution of Mn(II) ions is yellow in colour.

The correct statements are:

(1) (I), (III) and (IV) only (2) (I), (II) and (III) only

(3) (II), (III) and (IV) only (4) (I) and (II) only

Sol. **2**

Mn^{2+} $[Ar]3d^5$ it can form low spin as well as high spin complex depending upon nature of ligand same of Ni^{2+} ion with coordination no 4. It can be dsp^2 or sp^3 i:e low spin or high spin depending open nature of ligand.

5. The statement that is not true about ozone is:

(1) in the stratosphere, it forms a protective shield against UV radiation.

(2) in the atmosphere, it is depleted by CFCs.

(3) in the stratosphere, CFCs release chlorine free radicals (Cl) which reacts with O_3 to give chlorine dioxide radicals.

(4) it is a toxic gas and its reaction with NO gives NO_2.

Sol. **3**

$$\dot{Cl} + O_3 \longrightarrow Cl\dot{O} + O_2$$

Chlorine monoxide

Hence option (3)

6. Consider the following reactions:

(i) $\text{Glucose} + \text{ROH} \xrightarrow{\text{dry HCl}} \text{Acetal} \xrightarrow[(CH_3CO)_2O]{\text{x eq. of}} \text{acetyl derivative}$

(ii) $\text{Glucose} \xrightarrow{Ni/H_2} \text{A} \xrightarrow[(CH_3CO)_2O]{\text{y eq. of}} \text{acetyl derivative}$

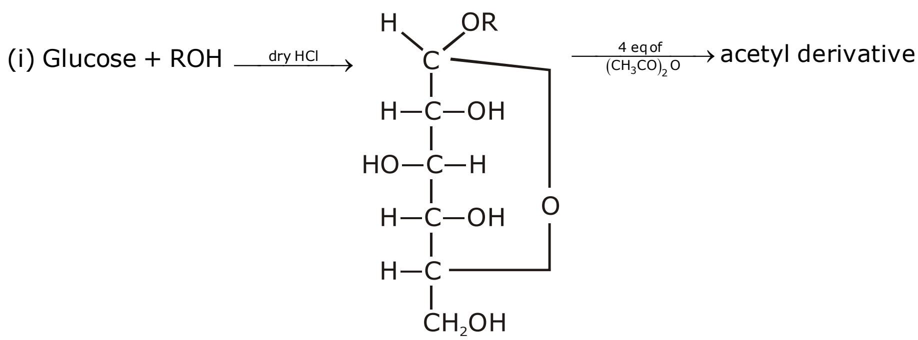
(iii) $\text{Glucose} \xrightarrow[(CH_3CO)_2O]{\text{z eq. of}} \text{acetyl derivative}$

'x', 'y' and 'z' in these reactions are respectively.

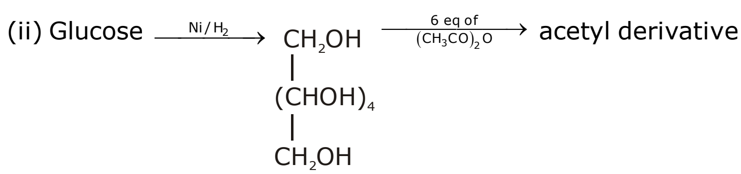
(1) 4, 5 & 5 (2) 5, 4 & 5

(3) 5, 6 & 5 (4) 4, 6 & 5

Sol. **4**



α–& β-alkyl
Glucose



(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{\text{5 eq. of}}$ Acetyl derivative

$(CH_3CO)_2O$ reacts with –OH group to form acetyl derivative, so as the no. of –OH group no. of eq. of $(CH_3CO)_2O$ will be used

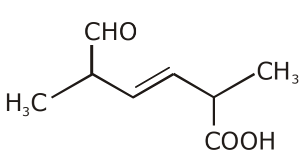
So, x = 4

y = 6

z = 5

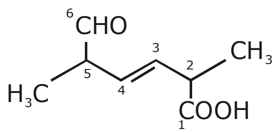
So, option (4) will be correct answer.

7. The IUPAC name for the following compound is:



(1) 2,5-dimethyl-5-carboxy-hex-3-enal
(2) 2,5-dimethyl-6-oxo-hex-3-enoic acid
(3) 6-formyl-2-methyl-hex-3-enoic acid
(4) 2,5-dimethyl-6-carboxy-hex-3-enal

Sol. **2**



2,5–Dimethyl–6–oxohex–3–enoic acid

8. For the following Assertion and Reason, the correct option is

Assertion (A): When Cu (II) and sulphide ions are mixed, they react together extremely quickly to give a solid.

Reason (R): The equilibrium constant of Cu^{2+} (aq) + S^{2-} (aq) $\rightleftharpoons$ CuS (s) is high because the solubility product is low.

(1) **(A)** is false and **(R)** is true.

(2) Both **(A)** and **(R)** are false.

(3) Both **(A)** and **(R)** are true but **(R)** is not the explanation for **(A)**.

(4) Both **(A)** and **(R)** are true but **(R)** is the explanation for **(A)**.

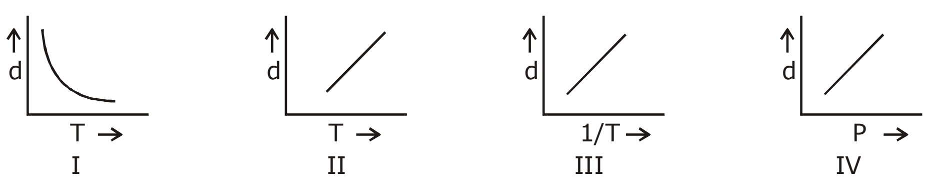
Sol. **4**

(A) is (B) true &

(R) is correct explanation of (A)

Ans. 4

9. Which one of the following graphs is not correct for ideal gas?



d = Density, P = Pressure, T = Temperature

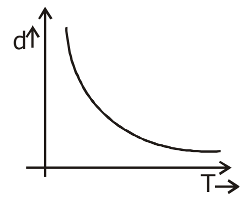
(1) I (2) IV (3) III (4) II

Sol. **4**

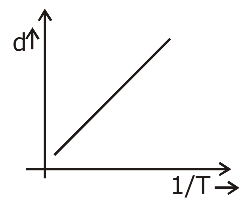
For ideal Gas

$$d = \frac{P \times M}{RT}$$

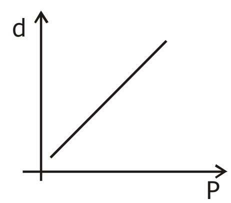
d v/s T $\rightarrow$ Hyperbolic



d v/s $\frac{1}{T} \rightarrow$ St. line



d v/s p $\rightarrow$ St line



$\therefore$ 'II' Graph is incorrect

Ans (4)

10. While titrating dilute HCl solution with aqueous NaOH, which of the following will not be required?

(1) Bunsen burner and measuring cylinder
(2) Burette and porcelain tile
(3) Clamp and phenolphthalein
(4) Pipette and distilled water

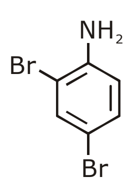
Sol. **1**

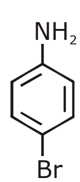
Bunsen Burner & measuring cylinder are not Required. As titration is already on exothermic pro cess

Ans.(1)

11. In Carius method of estimation of halogen, 0.172 g of an organic compound showed presence of 0.08 g of bromine. Which of these is the correct structure of the compound?

(1) $H_3C–Br$

(2) 

(3) 

(4) $H_3C–CH_2–Br$

Sol. **3**

carius method

mass % of 'Br' = $\frac{0.08}{0.172} \times 100 = \frac{8000}{172} = 46.51\%$

option (1) mass % = $\frac{80}{95} \times 100$

(2) mass % = $\frac{2 \times 80 \times 100}{252}$

(3) mass % = $\frac{1 \times 80 \times 100}{80 + 72 + 6 + 14} = \frac{8000}{172}\%$

(4) mass % = $\frac{1 \times 80 \times 100}{109}\%$

Option (3) matches with the given mass percentage value

Ans (3)

12. On heating compound (A) gives a gas (B) which is a constituent of air. This gas when treated with H_2 in the presence of a catalyst gives another gas (C) which is basic in nature. (A) should not be:

(1) $(NH_4)_2Cr_2O_7$ (2) NaN_3 (3) NH_4NO_2 (4) $Pb(NO_3)_2$

Sol. **4**

The gas (B) is N_2 which is found in air

$N_2 + 3H_2 \underset{}{\overset{Fe/Mo}{\rightleftharpoons}} 2NH_3$ (Haber's process)

(Basic in nature)

$NH_3 + H_2O \rightarrow NH_4OH$ (weak base)

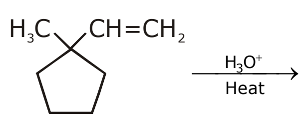
$(NH_4)_2Cr_2O_7 \longrightarrow N_2 + Cr_2O_3 + H_2O$

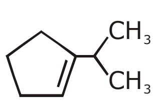
$NaN_3 \longrightarrow N_2 + Na$

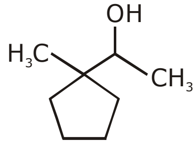
$NH_4NO_2 \longrightarrow N_2 + H_2O$

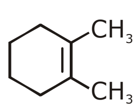
$Pb(NO_3)_2 \longrightarrow PbO + NO_2 + O_2$

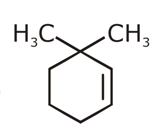
13. The major product in the following reaction is:



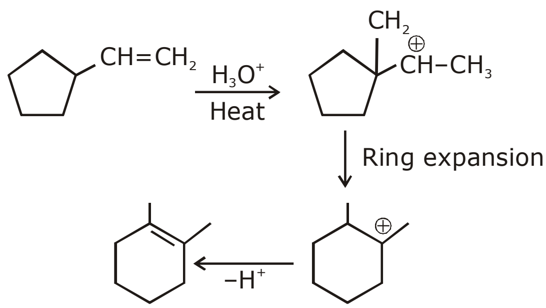
(1) 

(2) 

(3) 

(4) 

Sol. **3**



Option (3) is correct answer.

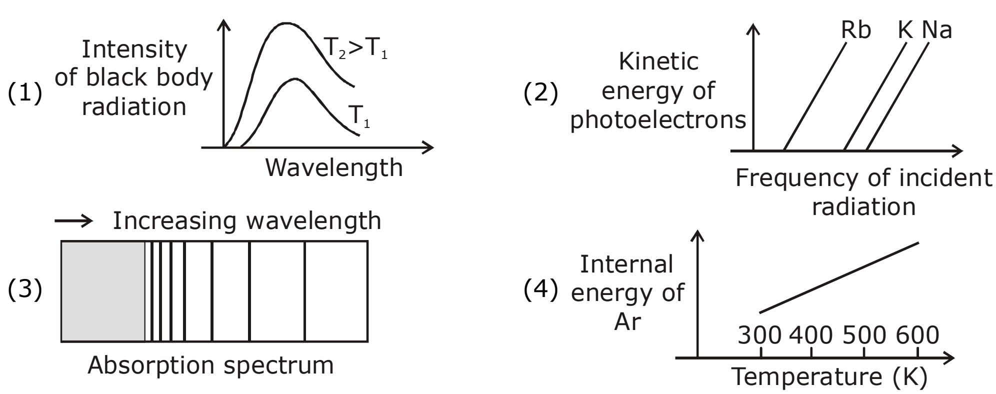
14. In general, the property (magnitudes only) that shows an opposite trend in comparison to other properties across a period is:

(1) Ionization enthalpy
(2) Electronegativity
(3) Atomic radius
(4) Electron gain enthalpy

Sol. **3**

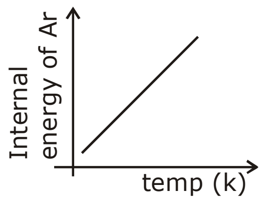
Ionisation energy, electronegativity & electron gain enthalpy increase across a period but atomic radius decreases

15. The figure that is not a direct manifestation of the quantum nature of atoms is:



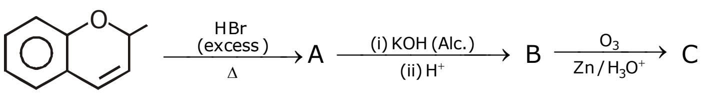
Sol. **4**

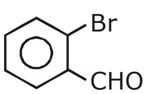
Internal energy of 'Ar' or any gas, has nothing to do with Quantum nature of atom hence

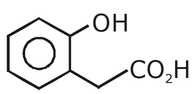


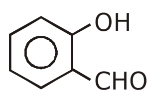
Ans. option (4)

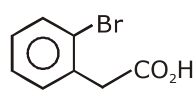
16. The major aromatic product C in the following reaction sequence will be :



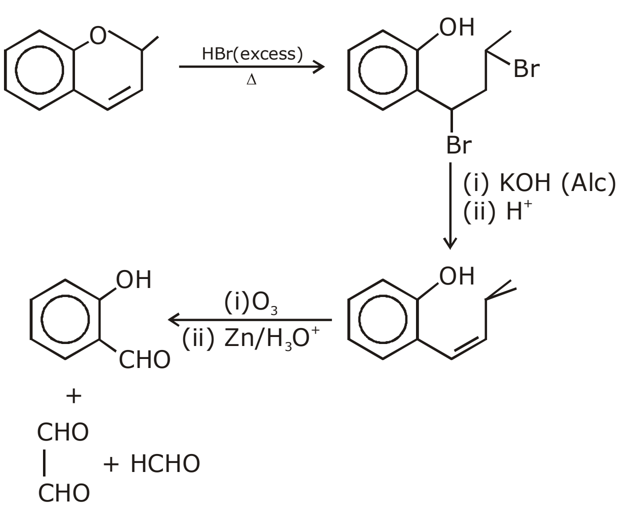
(1) 

(2) 

(3) 

(4) 

Sol. **3**



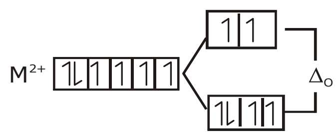
Option (3) is correct answser.

17. Consider that a d^6 metal ion (M^{2+}) forms a complex with aqua ligands, and the spin only magnetic moment of the complex is 4.90 BM. The geometry and the crystal field stabilization energy of the complex is:

(1) tetrahedral and $-0.6\Delta_t$ (2) tetrahedral and $-1.6\Delta_t + 1P$

(3) octahedral and $-1.6\Delta_0$ (4) octahedral and $-2.4\Delta_0 + 2P$

Sol. **1**



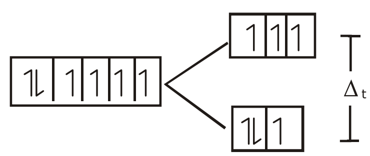
μ spin = 4.9 BM

CFSE = $-0.4 \times 4\Delta_0 + 0.6 \times 2\Delta_0$

= $[1.6 + 1.2]\Delta_0$

= $-0.4\ \Delta_0$

$\Rightarrow$



CFSE = $-0.6 \times 3\Delta_t + 0.4 \times 3\Delta_t$

= $-1.8\ \Delta_t + 1.2\Delta_t$

= $-0.6\ \Delta_t$

18. If AB_4 molecule is a polar molecule, a possible geometry of AB_4 is:

(1) Square planar (2) Tetrahedral

(3) Square pyramidal (4) Rectangular planar

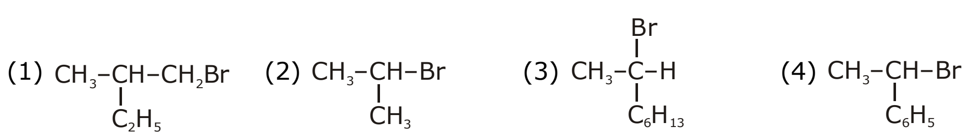
Sol. 1

Incorrect question Option 1 is more appropriate with respect to given option

(Chemical bonding)

(Options are incorrect)

19. Which of the following compounds will show retention in configuration on nucleophilic substitution by OH^- ion?



Sol. **1**

In $CH_3-\underset{\underset{C_2H_5}{|}}{CH}-CH_2Br$ attack of OH^- is not on chiral carbon, it is adjacent to chiral carbon, so configuration of chiral carbon remains constant.

20. The metal mainly used in devising photoelectric cells is:

(1) Li (2) Cs (3) Rb (4) Na

Sol. **2**

'Cs' is used in photoelectric cell as its ionisation energy is lowest

Hence Ans (2)

21. The mass of gas adsorbed, x, per unit mass of adsorbate, m, was measured at various pressures, p. A graph between $\log\frac{x}{m}$ and log p gives a straight line with slope equal to 2 and the intercept equal to 0.4771. The value of $\frac{x}{m}$ at a pressure of 4 atm is: (Given log3 = 0.4771)

Sol. 48

$\frac{x}{m} = KP^{1/n}$

$\log\,(x/m) = \log_{(k)} + \frac{1}{n}\log(p)$

$y = c + mx$

Intercept $C = \log_k = 0.4771$

slop $= \frac{1}{n} = 2$, $k = 3$

$\frac{x}{m} = k(P)^{1/n}$ at P = 4 atm

$= 3(4)^2$

$\frac{x}{m} = 3 \times 16 = 48$ Ans

22. The Gibbs energy change (in J) for the given reaction at $[Cu^{2+}] = [Sn^{2+}] = 1$ M and 298 K is:

$Cu(s) + Sn^{2+}(aq.) \rightarrow Cu^{2+}(aq.) + Sn(s)$

($E^o_{Sn^{2+}|Sn} = -0.16V$, $E^o_{Cu^{2+}|Cu} = 0.34V$, Take $F = 96500\ C\ mol^{-1}$)

Sol. **96500**

$Cu(s) + Sn^{+2}(aq) \rightleftharpoons Cu^{+2}\ (aq) + Sn(s)$

$E^o_{cell} = -0.16 - 0.34$

$= -0.50$

$\Delta G^o = -nF\,E^o_{cell}$

$= -2 \times 96500 \times (-0.5)$

$= +96500$

$\Delta G = \Delta G^o + RT\ \ell nQ$

$= 96500 + \frac{25}{3} \times 298 \times 2.303 \log(1)$

$\Delta G = 96500$ Joules

23. The internal energy change (in J) when 90 g of water undergoes complete evaporation at 100º C is __________.

(Given: ΔH_{vap} for water at 373 K = 41 kJ/mol, R = 8.314 JK^{-1} mol^{-1})

Sol. $H_2O\ (\ell) \longrightarrow H_2O\ (g)$

$\Delta E_{vap} = \Delta H_{vap} - \Delta ngRT$

$= 41000 \times 5 - 5 \times 8.314 \times 373$

$= 189494.39$

24. The oxidation states of iron atoms in compounds (A), (B) and (C), respectively, are x, y and z. The sum of x, y and z is ______.

$Na_4[Fe(CN)_5(NOS)]$	$Na_4[FeO_4]$	$[Fe_2(CO)_9]$
(A)	(B)	(C)

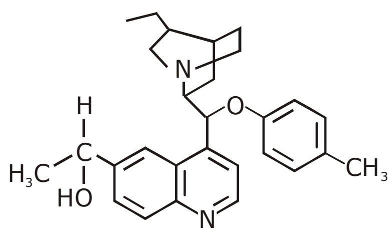
Sol. **6**

$Na_4\ [Fe^{+2}(CN)_5(NOS)]$

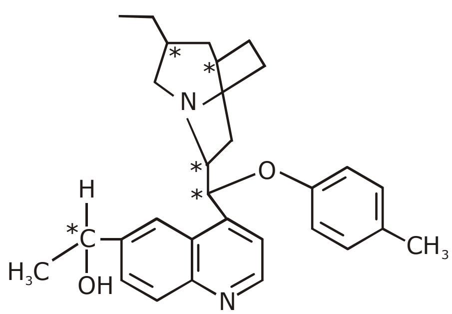
$Na_4[Fe^{+4}O_4]$

$[Fe_2^{\ 0}(CO)_9]$

25. The number of chiral carbons present in the molecule given below is _________.



Sol. **5**



Total chiral carbon = 5

QUESTION PAPER WITH SOLUTION

MATHEMATICS - 2 Sep 2020 - SHIFT - 1

Q.1 A line parallel to the straight line 2x-y=0 is tangent to the hyperbola $\frac{x^2}{4}-\frac{y^2}{2}=1$ at the point (x_1,y_1). Then $x_1^2+5y_1^2$ is equal to :

(1) 6 (2) 10 (3) 8 (4) 5

Sol. **1**

$T : \frac{xx_1}{4} - \frac{yy_1}{2} = 1$ ....(1)

t : 2x — y = 0 is parallel to T

$\Rightarrow T : 2x - y = \lambda$ ....(2)

Now compare (1) & (2)

$\frac{\frac{x_1}{4}}{2} = \frac{\frac{y_1}{2}}{1} = \frac{1}{\lambda}$

$x_1 = 8/\lambda$ & $y_1 = 2/\lambda$

(x_1,y_1) lies on hyperbola $\Rightarrow \frac{64}{4\lambda^2} - \frac{4}{2\lambda^2} = 1$

$\Rightarrow 14 = \lambda^2$

Now $= x_1^2 + 5y_1^2$

$= \frac{64}{\lambda_2} + 5\frac{4}{\lambda_2}$

$= \frac{84}{14}$

= 6 Ans.

Q.2 The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty,-a]\cup[a,\infty)$. Then a is equal to :

(1) $\frac{\sqrt{17}-1}{2}$ (2) $\frac{\sqrt{17}}{2}$ (3) $\frac{1+\sqrt{17}}{2}$ (4) $\frac{\sqrt{17}}{2}+1$

Sol. **3**

$-1 \le \frac{|x|+5}{x^2+1} \le 1$

$-x^2-1 \le |x|+5 \le x^2+1$

case - I

$-x^2-1 \le |x|+5$

this inequality is always right $\forall\ x \in R$

case - II

$|x|+5 \le x^2+1$

$x^2 - |x| \ge 4$

$|x|^2 - |x| - 4 \geq 0$

$$\left(|x| - \left(\frac{1+\sqrt{17}}{2}\right)\right)\left(|x| - \left(\frac{1-\sqrt{17}}{2}\right)\right) \geq 0$$

$|x| \leq \frac{1-\sqrt{17}}{2} \cup |x| \geq \frac{1+\sqrt{17}}{2}$

not possible

$$x \in \left(-\infty, \frac{-1-\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$$

$a = \frac{1+\sqrt{17}}{2}$

Q.3 If a function f(x) defined by $f(x) = \begin{cases} ae^x + be^{-x}, & -1 \leq x < 1 \\ cx^2, & 1 \leq x \leq 3 \\ ax^2 + 2cx, & 3 < x \leq 4 \end{cases}$ be continuous for some a, b,c $\in$ R and

f′(0)+f′(2) =e, then the value of a is :

(1) $\frac{1}{e^2 - 3e + 13}$ (2) $\frac{e}{e^2 - 3e - 13}$ (3) $\frac{e}{e^2 + 3e + 13}$ (4) $\frac{e}{e^2 - 3e + 13}$

Sol. **4**

f(x) is continuous

at x=1 $\Rightarrow \boxed{ae + \frac{b}{e} = c}$

at x=3 $\Rightarrow$ 9c = 9a + 6c $\Rightarrow$ c=3a

Now $f'(0) + f'(2) = e$

$\Rightarrow$ a − b + 4c = e

$\Rightarrow$ a − e (3a−ae) + 4.3a = e

$\Rightarrow$ a − 3ae + ae^2 + 12a = e

$\Rightarrow$ 13a − 3ae + ae^2=e

$$\Rightarrow \boxed{a = \frac{e}{13 - 3e + e^2}}$$

Q.4 The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

(1) $(-\infty, -9] \cup [3, \infty)$ (2) $[-3, \infty)$ (3) $(-\infty, 9]$ (4) $(-\infty, -3] \cup [9, \infty)$

Sol. **4**

$\frac{a}{r}.a.ar = 27 \Rightarrow a = 3$

$\frac{a}{r}+a+ar=S$

$\frac{1}{r}+1+r=\frac{S}{3}$

$r+\frac{1}{r}=\frac{S}{3}-1$

$r+\frac{1}{r}\geq 2$ or $r+\frac{1}{r}\leq -2$

$\frac{S}{3}\geq 3$ or $\frac{S}{3}\leq -1$

$S\geq 9$ or $S\leq -3$

$S\in(-\infty,-3]\cup[9,\infty)$

Q.5 If $R=\{(x,y):x,y\in Z, x^2+3y^2\leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

(1) $\{-1,0,1\}$ (2) $\{-2,-1,1,2\}$ (3) $\{0,1\}$ (4) $\{-2,-1,0,1,2\}$

Sol. **1**

$3y^2\leq 8-x^2$

R : {(0,1), (0,−1), (1,0),(−1,0), (1,1), (1,-1)
(-1,1), (-1,-1), (2,0), (-2,0),(-2,0),(2,1),(2,-1), (-2,1),(-2,-1)}

$\Rightarrow$ R : {-2,-1,0,1,2}→{-1,0,-1}

Hence R^{-1} : {-1,0,1}→{-2,-1,0,1,2}

Q.6 The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :

(1) $-\frac{1}{2}(1-i\sqrt{3})$ (2) $\frac{1}{2}(1-i\sqrt{3})$ (3) $-\frac{1}{2}(\sqrt{3}-i)$ (4) $\frac{1}{2}(\sqrt{3}-i)$

Sol. **3**

$$\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$$

$$=\left(\frac{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)+i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}{1+\cos\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)-i\sin\left(\frac{\pi}{2}-\frac{2\pi}{9}\right)}\right)^3$$

$$= \left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$= \left(\frac{2\cos\frac{5\pi}{36}\left\{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right\}}{2\cos\frac{5\pi}{36}\left\{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}\right\}}\right)^3$$

$$= \left(\frac{\text{cis}\left(\frac{5\pi}{36}\right)}{\text{cis}\left(\frac{-5\pi}{36}\right)}\right)^3$$

$$= \text{cis}\left(\frac{5\pi}{36}\times 3+\frac{5\pi}{36}\times 3\right)$$

$$= \text{cis}\left(\frac{10\pi}{12}\right)$$

$$= \text{cis}\left(\frac{5\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}+\frac{i}{2}}$$

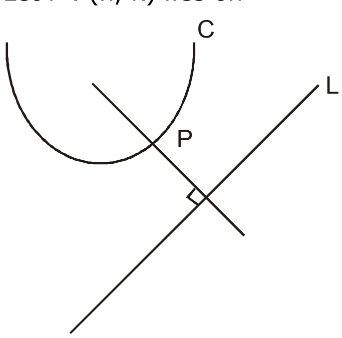
Q.7 Let P(h,k) be a point on the curve $y=x^2+7x+2$, nearest to the line, $y=3x-3$. Then the equation of the normal to the curve at P is:

(1) $x+3y-62=0$ (2) $x-3y-11=0$ (3) $x-3y+22=0$ (4) $x+3y+26=0$

Sol. **4**

C : $y = x^2 + 7x+2$

Let P : (h, k) lies on



Curve = $k = h^2+7h+2$

Now for shortest distance

$M_T |_p^c = m_L = 2h+7 = 3$
h =-2
k=-8
P : (-2,-8)
equation of normal to the curve is perpendicular to L : 3x – y = 3
N : x + 3y = λ
$\downarrow$ Pass (-2,-8)
λ = -26
N : x + 3y + 26 = 0

Q.8 Let A be a 2×2 real matrix with entries from $\{0,1\}$ and $|A| \neq 0$. Consider the following two statements:
(P) If $A \neq I_2$, then $|A| = -1$
(Q) If |A|=1, then tr(A) =2,
where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then:
(1) Both (P) and (Q) are false
(2) (P) is true and (Q) is false
(3) Both (P) and (Q) are true
(4) (P) is false and (Q) is true

Sol. **4**

$P : A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \neq I_2$ & $|A| \neq 0$ & $|A| = 1$(false)

$Q : A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = 1$ then Tr(A) = 2 (true)

Q.9 Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

(1) $\frac{4}{17}$ (2) $\frac{8}{17}$ (3) $\frac{2}{5}$ (4) $\frac{2}{3}$

Sol. **2**

$\boxed{1 \text{ to } 30}$
box I

Prime on I
{2,3,5,7,11,13,17,19,23,29}

$\boxed{31 \text{ to } 50}$
box II

Prime on II
{31,37,41,43,47}
A : selected number on card is non - prime
P(A) = P(I).P(A/I) + P(II). P(A/II)

$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \cdot \frac{15}{20}$

Now, P(I/A) = $\frac{P(II).P(A/I)}{P(A)}$

$$= \frac{\frac{1}{2} \cdot \frac{20}{30}}{\frac{1}{2} \cdot \frac{20}{30} + \frac{1}{2} \cdot \frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{3}{4}} = \frac{8}{17}$$

Q.10 If p(x) be a polynomial of degree three that has a local maximum value 8 at x=1 and a local minimum value 4 at x=2; then p(0) is equal to :

(1) 12 (2) -12 (3) -24 (4) 6

Sol. **2**

$p'(1) = 0$ & $p'(2) = 0$

$p'(x) = a(x-1)(x-2)$

$$p(x) = a\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + b$$

$$p(1) = 8 \Rightarrow a\left(\frac{1}{3} - \frac{3}{2} + 2\right) + b = 8 \quad \text{...(i)}$$

$$p(2) = 4 \Rightarrow a\left(\frac{8}{3} - \frac{3.4}{2} + 2.2\right) + b = 4 \quad \text{......(ii)}$$

from equation (i) and (ii)

$a = 24$ & $b = -12$

$p(0) = b = \boxed{-12}$

Q.11 The contrapositive of the statement "If I reach the station in time, then I will catch the train'' is:

(1) If I will catch the train, then I reach the station in time.
(2) If I do not reach the station in time, then I will catch the train.
(3) If I do not reach the station in time, then I will not catch the train.
(4) If I will not catch the train, then I do not reach the station in time.

Sol. **4**

Statement p and q are true

Statement, then the contra positive of the implication

$p \rightarrow q = (\sim q) \rightarrow (\sim p)$

hence correct Ans. is 4

Q.12 Let α and β be the roots of the equation, $5x^2+6x-2=0$. If $S_n = \alpha^n + \beta^n$, n=1,2,3,....., then:

(1) $5S_6+6S_5+2S_4=0$ (2) $6S_6+5S_5=2S_4$
(3) $6S_6+5S_5+2S_4=0$ (4) $5S_6+6S_5=2S_4$

Sol. **4**

$5x^2 + 6x - 2 = 0 <^{\alpha}_{\beta} = 5\alpha^2 + 6\alpha = 2$

$6\alpha - 2 = -5\alpha^2$

Simillarly

$6\beta - 2 = -5\beta^2$

$S_6 = \alpha^6 + \beta^6$

$S_5 = \alpha^5 + \beta^5$

$S_4 = \alpha^4 + \beta^4$

Now $6S_5 - 2S_4$

$= 6\alpha^5 - 2\alpha^4 + 6\beta^5 - 2\beta^4$

$= a^4(6\alpha-2) + \beta^4(6\beta-2)$

$= \alpha^4(-5\alpha^2) + \beta^4(-5\beta^2)$

$= -5(\alpha^6 + \beta^6)$

$= -5S_6$

$= 6S_5 + 5S_6 = 2S_4$

Q.13 If the tangent to the curve y=x+siny at a point (a,b) is parallel to the line joining $\left(0,\frac{3}{2}\right)$ and $\left(\frac{1}{2},2\right)$, then:

(1) $b = \frac{\pi}{2} + a$ (2) |a+b|=1 (3) |b-a|=1 (4) b=a

Sol. **3**

$$\left.\frac{dy}{dx}\right|^{c}_{p(a,b)} = \frac{2-\frac{3}{2}}{\frac{1}{2}-0}$$

$$\begin{array}{l|l} 1 + \cos b = 1 & p : (a,b) \text{ lies on curve} \\ \cos b = 0 & \quad b = a + \sin b \end{array}$$

$$\boxed{b = a \pm 1}$$

$$b - a = \pm 1$$

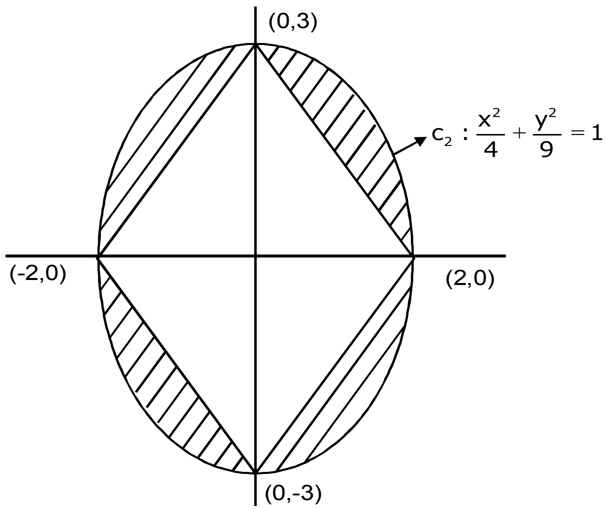
$$\boxed{|b - a| = 1}$$

Q.14 Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is:

(1) $3(\pi - 2)$ (2) $6(\pi - 2)$ (3) $6(4 - \pi)$ (4) $3(4 - \pi)$

Sol. **2**

$c_1 : \frac{|x|}{2} + \frac{|y|}{3} = 1$



$A = 4\left(\frac{\pi ab}{4} - \frac{1}{2}.2.3\right)$

$A = \pi$. 2. 3 – 12

$A = 6(\pi - 2)$

Q.15 If $|x|<1, |y|<1$ and $x \neq y$, then the sum to infinity of the following series $(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+....$ is:

(1) $\frac{x+y+xy}{(1-x)(1-y)}$ (2) $\frac{x+y-xy}{(1-x)(1-y)}$ (3) $\frac{x+y+xy}{(1+x)(1+y)}$ (4) $\frac{x+y-xy}{(1+x)(1+y)}$

Sol. **2**

$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+.... \infty$

$= \frac{1}{(x-y)}\left\{\left(x^2-y^2\right)+\left(x^3-y^3\right)+\left(x^4-y^4\right)+...\infty\right\}$

$= \frac{\frac{x^2}{1-x} - \frac{y^2}{1-y}}{x-y}$

$= \frac{x^2(1-y)-y^2(1-x)}{(1-x)(1-y)(x-y)}$

$= \frac{(x^2-y^2)-xy\ (x-y)}{(1-x)(1-y)(x-y)} = \frac{((x+y)-xy)(x-y)}{(1-x)(1-y)(x-y)}$

$= \frac{x+y-xy}{(1-x)(1-y)}$

Q.16 Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term indepen dent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to:

(1) 176 (2) 336 (3) 352 (4) 84

Sol. **2**

For term independent of x

$T_{r+1} = {}^{10}C_r \left(\alpha x^{\frac{1}{9}}\right)^{10-r} . \left(\beta x^{-\frac{1}{6}}\right)^{r}$

$T_{r+1} = {}^{10}C_r \alpha^{10-r} \beta^r . x^{\frac{10-r}{9}} . x^{-\frac{r}{6}}$

$\because \frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r = 4$

$T_5 = {}^{10}C_r \alpha^6 . \beta^4$

$\because$ AM $\geq$ GM

Now $\frac{\left(\frac{\alpha^3}{2} + \frac{\alpha^3}{2} + \frac{\beta^2}{2} + \frac{\beta^2}{2}\right)}{4} \geq \sqrt[4]{\frac{\alpha^6 . \beta^4}{2^4}}$

$\left(\frac{4}{4}\right)^4 \geq \frac{\alpha^6 \beta^4}{2^4}$

$\alpha^6 . \beta^4 \leq 2^4$

${}^{10}C_4 . \alpha^6 . \beta^4 \leq {}^{10}C_4 \, 2^4$

$T_5 \leq {}^{10}C_4 2^4$

$T_5 \leq \frac{10!}{6!4!} . 2^4$

$T_5 \leq \frac{10.9.8.7.2^4}{4.3.2.1}$

maximum value of T_5 = 10. 3.7. 16 = 10k

k = 16.7.3

k = 336

Q.17 Let S be the set of all $\lambda \in R$ for which the system of linear equations

2x-y+2z=2

x-2y+λz=-4

x+λy+z=4

has no solution. Then the set S

(1) is an empty set.
(2) is a singleton.
(3) contains more than two elements.
(4) contains exactly two elements.

Sol. **4**

For no solution

$\Delta = 0$ & $\Delta_1 | \Delta_2 | \Delta_3 \neq 0$

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$2(-2-\lambda^2) + 1(1-\lambda) + 2(\lambda+2) = 0$

$-4 - 2\lambda^2 + 1 - \lambda + 2\lambda + 4 = 0$

$-2\lambda^2 + \lambda + 1 = 0$

$2\lambda^2 - \lambda - 1 = 0 \Rightarrow \lambda = 1, -1/2$

Equation has exactly 2 solution

Q.18 Let $X = \{x \in N : 1 \leq x \leq 17\}$ and $Y = \{ax + b : x \in X \text{ and } a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then a+b is equal to:

(1)-27
(2) 7
(3)-7
(4) 9

Sol. **3**

X : {1,2,.....17}

Y : {ax+b : x$\in$X & a, $b \in R$, a>o}

Given Var(Y) = 216

$$\frac{\sum y_1^2}{n} - (\text{mean})^2 = 216$$

$$\frac{\sum y_1^2}{17} - 289 = 216$$

$\sum y_1 = 8585$

$(a+b)^2 + (2a+b)^2 + ... + (17a+b)^2 = 8585$

$105a^2 + b^2 + 18ab = 505$ ....(1)

Now $\sum y_1 = 17 \times 17$

$a(17\times9) + 17.b = 17 \times 17$

$9a + b = 17$ ....(2)

from equation (1) & (2)

a = 3 & b = -10

a+b = –7

Q.19 Let y=y(x) be the solution of the differential equation, $\frac{2+\sin x}{y+1}\cdot\frac{dy}{dx} = -\cos x, y > 0, y(0) = 1.$ If $y(\pi) = a$, and $\frac{dy}{dx}$ at $x = \pi$ is b, then the ordered pair (a,b) is equal to:

(1) $\left(2, \frac{3}{2}\right)$ (2) (1,1) (3) (2,1) (4) (1,-1)

Sol. **2**

$$\int \frac{dy}{y+1} = \int \frac{-\cos x\ dx}{2+\sin x}$$

ln |y+1| = -ln |2+sin x| + k

$\downarrow$ (0,1)

k = ln 4

Now C : (y+1) (2+sin x) = 4

$y(\pi)=a \Rightarrow (a+1)(2+0)=4 \Rightarrow (a=1)$

$$\left.\frac{dy}{dx}\right|_{x=\pi} = b \Rightarrow b = -(-1)\left(\frac{2+0}{1+1}\right)$$

$\Rightarrow$ b = 1

(a,b) = (1,1)

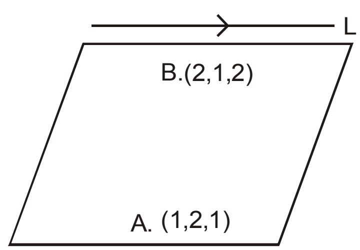
Q.20 The plane passing through the points (1,2,1), (2,1,2) and parallel to the line, 2x=3y, z=1 also passes through the point:

(1) (0,-6,2) (2) (0,6,-2) (3) (-2,0,1) (4) (2,0,-1)

Sol. **3**

$$L : \begin{cases} 2x = 3y \\ z = 1 \end{cases} < \begin{matrix} P:(0,0,1) \\ Q:(3,2,1) \end{matrix}$$

$\vec{V}_L$ Dr of line (3,2,0)



$\vec{n}_p = \overrightarrow{AB} \times \vec{V}_L$

$\vec{n}_p = \langle 1, -1, 1\rangle \times \langle 3, 2, 0\rangle$

$\vec{n}_p = \langle -2, +3, 5\rangle$

Plane : −2(x-1)+3(y-2)+5(z -1)=0

Plane : $-2x+3y+5z+2-6-5=0$
Plane : $2x - 3y - 5z = -9$

Q.21 The number of integral values of k for which the line, $3x+4y=k$ intersects the circle, $x^2+y^2-2x-4y+4=0$ at two distinct points is...............

Sol. **9**

c : (1,2) & r = 1

$|cp| < r$

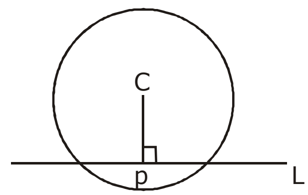
$$\left|\frac{3.1+4.2-k}{5}\right| < 1$$

$|11-k|<5$

$-5<k-11<5$

$6<k<16$

$k = 7, 8, 9, \ldots\ldots, 15 \Rightarrow$ total 9 value of k



Q.22 Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8$. Then $|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$ is equal to :

Sol. **2**

$$|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8$$

$$(\vec{a}-\vec{b}).(\vec{a}-\vec{b})+(\vec{a}-\vec{c})(\vec{a}-\vec{c})=8$$

$a^2+b^2 - 2a.b + a^2+c^2-2a.c=8$

$2a^2+b^2+c^2-2a.b - 2a.c=8$

$a.b + a.c=-2$

Now $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$

$= 2a^2+4b^2+4c^1+4\bar{a}.\bar{b}+4\bar{a}.\bar{c}$

$= 2 + 4 + 4 + 4(-2)$

$= 2$

Q.23 If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is.........

Sol. **309**

E H M O R T

E - - - - -	= 5!
H - - - - -	= 5!
M E - - - -	= 4!
M H - - - -	= 4!
M O E - - -	= 3!
M O H - - -	= 3!
M O R - - -	= 3!
M O T E - -	= 2!
M O T H E R	= 1
	309

Q.24. If $\lim_{x\to 1}\frac{x+x^2+x^3+...+x^n-n}{x-1}=820,(n\in N)$ then the value of n is equal to :

Sol. **40**

$$\lim_{x\to 1}\frac{(x-1)}{x-1}+\frac{(x^2-1)}{x-1}+....+\frac{(x^n-1)}{x-1}=820$$

$\Rightarrow$ 1 + 2 + 3 + ...+n = 820

$\Rightarrow \sum n=820$

$\Rightarrow \frac{n(n+1)}{2}=820$

$\Rightarrow$ n = 40

Q.25 The integral $\int_0^2 ||x-1|-x|\,dx$ is equal to :

Sol. **1.5**

$$\int_0^2 ||x-1|-x|\,dx$$

$$=\int_0^1 |1-x-x|\,dx+\int_1^2 |x-1-x|\,dx$$

$$=\int_0^1 |2x-1|\,dx+\int_1^2 1\,dx$$

$$=\int_0^{\frac{1}{2}}(1-2x)\,dx+\int_{\frac{1}{2}}^1 (2x-1)\,dx+\int_1^2 1\,dx$$

$$=\left[\left(\frac{1}{2}-0\right)-\left(\frac{1}{4}-0\right)\right]+\left(1-\frac{1}{4}\right)-\left(1-\frac{1}{2}\right)+1$$

$$=\frac{1}{2}-\frac{1}{4}+\frac{3}{4}-\frac{1}{2}+1$$

$$=\frac{3}{2}$$