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DISTRICT LEVEL II PUC PREPARATORY EXAM, MARCH - 2022

Time: 3 Hrs. 15 Mins. **Sub: MATHEMATICS (35)** **Max. Marks: 100**

General Instructions:

1. The question paper has Five parts, namely A, B, C, D and E. Answer all the parts.
2. Part – A carries 10 marks, Part – B carries 20 marks, Part – C carries 30 Marks, Part – D carries 30 marks and Part – E carries 10 marks.
3. Write the corresponding question number properly as indicated in the question paper.
4. Use the graph sheet for the questions on LPP in Part – E.

PART - A

I. **Answer any TEN of the following questions:** $10 \times 1 = 10$

1. A relation R on $A = \{1,2,3\}$ defined by $R = \{(1,1)(1,2)(3,3)\}$ is not symmetric. Why?
2. Define a binary operation.
3. Write the domain of $f(x) = \sin^{-1} x$.
4. Find the value of $\cos\left[\sec^{-1} x + \operatorname{cosec}^{-1} x\right], |x| \geq 1$.
5. Define scalar matrix.
6. Find the value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
7. If $y = \cos(1-x)$ find $\frac{dy}{dx}$.
8. If $y = \tan(2x+3)$ find $\frac{dy}{dx}$.
9. Find $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$.
10. Find $\int \operatorname{cosec} x(\operatorname{cosec} x + \cot x)dx$.
11. Find the unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$.
12. Define negative of a vector.
13. Write the direction cosines of y-axis.
14. Define feasible region of a linear programming problem.
15. Find $P(A/B)$; if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.

PART - B

II. **Answer any TEN of the following questions:** $10 \times 2 = 20$

16. If $f: R \to R$ defined by $f(x) = 1 + x^2$ then show that f is neither one-one nor onto.
17. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.
18. Write the simplest form of $\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$.
19. Find the values of x, y and z from the equation $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$.
20. Find area of the triangle whose vertices are $(-2,-3)$, $(3,2)$ and $(-1,-8)$ by using determinant.
21. If $y + \sin y = \cos x$ find $\frac{dy}{dx}$.

P.T.O.

22. Differentiate $(\sin x)^x$ with respect to x.

23. If $x = at^2,\ y = 2at$ find $\frac{dy}{dx}$.

24. Find the slope of tangent to the curve $y = \frac{x-1}{x-2}$ $at\ x = 10$.

25. Evaluate $\int \sin x.\sin(\cos x)dx$

26. Find $\int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x}dx$.

27. Evaluate $\int_2^3 \frac{x}{x^2+1}dx$.

28. Find the order and degree of the differential equation $y''' + y^2 + e^{y'} = 0$.

29. If $\vec{a}$ is a unit vector such that $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8$ find $|\vec{x}|$.

30. Find the area of parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

31. Find the angle between pairs of lines

$\vec{r} = 2\hat{\imath} - 5\hat{\jmath} + \hat{k} + \lambda(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ and $\vec{r} = 7\hat{\imath} - 6\hat{k} + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

32. Find the distance of the point $(-6, 0, 0)$ from the plane $2x - 3y + 6z = 0$.

33. A random variable X has the following probability distribution:

X	0	1	2	3	4
P(X)	0.1	K	2K	2K	K

Determine K.

PART - C

III. **Answer any TEN of the following questions:** **10 × 3 = 30**

34. Show that the relation R in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b)/|a-b|\ is\ even\}$ is an equivalence relation.

35. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

36. Express $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as sum of symmetric and skew-symmetric matrices.

37. Verify that if any two rows of a determinant are interchanged then the sign of determinant changes by considering $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

38. If $y^x = x^y$ then find $\frac{dy}{dx}$.

39. If $x = a[\theta + \sin\theta]$ and $y = a[1 - \cos\theta]$ prove that $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$.

40. Verify mean value theorem if $f(x) = x^3 - 5x^2 - 3x\ \forall x \in [1,3]$

P.T.O.

41. Find the intervals in which the f is given by $f(x) = x^2 - 4x + 6$ is (a) Strictly increasing (b) strictly decreasing.

42. Find $\int e^x \left[\frac{1+\sin x}{1+\cos x}\right] dx$.

43. Evaluate $\int \frac{x}{(x-1)(x-2)} dx$.

44. Evaluate $\int_0^2 e^x dx$ as the limit of a sum.

45. Find the area of the region bounded by the curve, $y = x^2$ and the line $y = 4$.

46. Form the differential equation of the family of curves $y = ae^{3x} + be^{-2x}$ by eliminating arbitrary constants a and b.

47. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

48. Prove that $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$

49. Show that the position vector of the point P which divides the line joining the points A and B having position vectors $\vec{a}$ *and* $\vec{b}$ internally in the ratio $m:n$ *is* $\frac{m\vec{b}+n\vec{a}}{m+n}$.

50. Find the shortest distance between the lines whose vector equations are $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$.

51. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART - D

IV. **Answer any SIX of the following questions:** 6 x 5 =30

52. Let the function $f: R \to R$ where R is the set of real number defined by $f(x) = 3 - 4x$. Is f one-one and onto? Justify your answer.

53. Let $f: N \to R$ be a function defined by $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$ where S is the range of f is invertible. Find the inverse of f.

54. If $A = \begin{bmatrix} 0 & 6 & 7 \\ 6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, calculate AC, BC and (A+B)C and also verify that $(A+B)C = AC + BC$.

55. Solve the system of linear equations by using matrix method: $2x + 3y + 3z = 5$, $x - 2y + z = -4$ and $3x - y - 2z = 3$

56. If $y = 3\cos(logx) + 4\sin(logx)$ show that $x^2y_2 + xy_1 + y = 0$.

57. If length x of a rectangle is decreasing at the rate of 3cm/minute and the width y is increasing at the rate of 2 cm/minute. When $x = 10cm$ and $y = 6cm$, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

P.T.O.

58. Find the integral of $\int \frac{1}{\sqrt{x^2-a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{4x^2-9}}dx$.

59. Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ using integration.

60. Find the general solution of the differential equation $\frac{dy}{dx}+y\sec x=\tan x,\ 0\le x\le\frac{\pi}{2}$.

61. Derive the equation of a plane in normal form both in the vector and Cartesian form.

62. A die is thrown 6 times. If getting an odd number is success what is the probability of (a) 5 success (b) at least 5 success (c) atmost 5 success.

63. Probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently then find the probability that (a) the problem is solved (b) Exactly one of them solves the problem.

PART - E

V. **Answer any ONE of the following questions:**

1 × 10 = 10

64. (a) Solve the following problem graphically. Maximize and minimize $Z = 10500x + 9000y$, subject to the constraints: $x + y \le 50,\ 2x + y \le 80,\ x \ge 0,\ y \ge 0$. [6M]

(b) Show that the matrix $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ satisfies the equation $A^2-4A+I=0$, when I is 2×2 identity matrix and 0 is 2×2 zero matrix. Using this equation, find A^{-1}. [4M]

65. (a) Prove that $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ and hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{1+\sqrt{\tan x}}dx$. [6M]

(b) Find K. If $f(x)=\begin{cases}\frac{K\cos x}{\pi-2x} & if\ x\ne\frac{\pi}{2}\\ 3 & if\ x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$. [4M]

66. (a) A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m. [6M]

(b) By using properties of determinants show that

$$\begin{vmatrix}x+y+2z & x & y\\ z & y+z+2x & y\\ z & x & z+x+2y\end{vmatrix}=2(x+y+z)^3$$ [4M]

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