SECOND PUC PREPARATORY EXAMINATION, MARCH - 2022

SUBJECT : MATHEMATICS (35)

Time : 3 Hrs. 15 Mins. **Max Marks : 100**

INSTRUCTIONS :

1) The question paper has five parts namely A, B, C, D and E. Answer all the parts.
2) Use the graph sheet for the question on Linear Programming problem in Part-E

PART - A

I Answer any TEN questions : **10x1=10**

1) The relation R on set A = {1, 2, 3} is defined as R = {(1, 1), (2, 2), (3, 3)} is not symmetric, why ?
2) Let $*$ be the binary operation on N given by $a*b$ = L.C.M. of 'a' and 'b'. Find $5*7$.
3) Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$.
4) If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$ then find the value of x.
5) Define column matrix.
6) If A is a square matrix order 2 with $|A| = 8$, then find $|AA^1|$.
7) Find $\frac{dy}{dx}$. If $y = \cos\sqrt{x}$
8) If $y = e^{\cos x}$ find $\frac{dy}{dx}$.
9) Find $\int\left(2x-3\cos x+e^x\right)dx$
10) Evaluate : $\int_4^5 e^x\,dx$.
11) Find the unit vector in the direction of the vector $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$.
12) Define collinear vectors.
13) Write the direction cosines Z-axis.
14) Define optimal solution in the linear programming problem.
15) If P(A) = 0.8, P(B) = 0.5 and $P\left(\frac{B}{A}\right)=0.4$ find $P(A\cap B)$.

PART -B

II Answer any TEN questions : **10x2=20**

16) Show that the Signum function $f(x)=\begin{cases}1 & \text{if } x>0\\ 0 & \text{if } x=0\\ -1 & \text{if } x<0\end{cases}$ is neither one-one nor onto.
17) Prove that $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, $x\in[-1,1]$.
18) Find the domain and range of $y = \sec^{-1}x$.
19) Find the values of x, y and z from the equation $\begin{bmatrix}4 & 3\\ x & 5\end{bmatrix}=\begin{bmatrix}y & z\\ 1 & 5\end{bmatrix}$.

(P.T.O.)

20) If the area of the triangle whose vertices are (-2, 0), (0, 4) and (0, k) if 4 sq units then find the value of k using determinants.

21) If $y + \sin y = \cos x$ find $\frac{dy}{dx}$.

22) If $x = 2at^2$, $y = at^4$ then find $\frac{dy}{dx}$.

23) Differentiate $\sec\left(\tan\sqrt{x}\right)$ with respect to 'x'.

24) Find the slope the tangent to the curve $y = \frac{x-1}{x-2}(x \neq 2)$ at $x = 10$.

25) Find $\int \frac{x^2}{1-x^6}\,dx$.

26) Find $\int \sin 2x \cdot \cos 3x\,dx$.

27) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

28) Find the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$.

29) Find the angle 'θ' between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

30) Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

31) Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

32) Find the distance of a point (2, 3, -5) from the plane $x + 2y - 2z = 9$.

33) The random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine k.

PART - C

III Answer any TEN questions : **10x3=30**

34) Show that the relation R in the set Z of integers given by R = {(x, y) : 2 divides x-y} is an equivalence relation.

35) Solve for x, $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ (where $x > 0$).

36) By using elementary transformation find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.

37) Verify that the value of the determinant remains unchanged if it's rows and columns are interchanged by considering third order determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

38) If $x\sqrt{1+y}+y\sqrt{1+x}=0$ for $-1<x<1$ and $x\neq y$ prove that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$

39) If $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$ find $\frac{dy}{dx}$

40) Verify Rolle's theorem for the function $f(x)=x^2+2x-8$, $x\in[-4,2]$

41) Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is
a) Strictly increasing b) Strictly decreasing.

42) Evaluate $\int\frac{(x-3)e^x}{(x-1)^3}dx$

43) Evaluate $\int x\log x\,dx$

44) Evaluate $\int_0^2 e^x\,dx$ as a limit of sum

45) Find the area of the region bounded by the curve $y^2=4x$ and line $x=3$

46) Form the differential equation of the family of circles touching the x-axis at origin

47) Solve $y\log y\,dx-x\,dy=0$

48) Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

49) Find x such that the four points A(3, 2, 1), B(4, x, 5), C(4, 2, -2) and D(6, 5, -1) are Co-planar.

50) Find the equation of the plane through the intersection of the planes
$3x-y+2z-4=0$ and $x+y+z-2=0$ and the point (2, 2, 1).

51) A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is 6. Find the probability that it is actually 6.

PART - D

IV Answer any SIX of the following questions : 6x5=30

52) Verify whether the function, $f: N\to N$ defined by $f(x)=x^2$ is one-one, onto and bijective.

53) Let $f: N\to R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f: N\to S$, where S is the range of function f is invertible. Find the inverse of f.

54) If $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ prove that $A^3-6A^2+7A+2I=0$.

55) Solve the following system of equations by matrix method :
$3x-2y+3z=8$; $2x+y-z=1$; $4x-3y+2z=4$.

56) If $y=Ae^{mx}+Be^{nx}$, show that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$

57) A ladder 24ft long leans against a vertical wall. The lower end is moving away at the rate of 3ft/sec. Find the rate at which the top of the ladder is moving downwards. If it's foot is 8ft from the wall.

58). Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ with respect to 'x' and hence evaluate $\int \frac{1}{\sqrt{x^2+7}}dx$.

59) Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (where a > b) by integration method.

60) Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2$, $(x \neq 0)$.

61) Derive the equation of a line in space passing through two given points both in Vector and Cartesian form.

62). A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$ what is the probability that he will win a prize

a) atleast once. b) exactly once.

63) If a fair coin is tossed 10 times, find the probability of

i) exactly six heads ii) atleast six heads.

PART - E

V Answer any ONE question : **1x10=10**

64) a) Solve the following linear programming problem graphically

Minimise and maximise $Z = x + 2y$ subject to the constraints

$x+2y \geq 100,\ 2x-y \leq 0,\ 2x+y \leq 200$ and $x, y \geq 0$. **(6)**

b) If the matrix $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$, where I is 2×2 identity matrix and '0' is a 2×2 zero matrix, using this equation find A^{-1}. **(4)**

65) a) Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function}\end{cases}$

and hence evaluate $\int_{-\pi/2}^{\pi/2}\left(x^3+x\cos x\right)dx$. **(6)**

b) Find the value of k so that the function

$$f(x)=\begin{cases}\frac{k\cos x}{\pi-2x} & \text{if } x \neq \frac{\pi}{2}\\ 3 & \text{if } x=\frac{\pi}{2}\end{cases}$$ is continuous at $x=\frac{\pi}{2}$. **(4)**

66) a) Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. **(6)**

b) Prove that $\begin{vmatrix}a^2+1 & ab & ac\\ ab & b^2+1 & bc\\ ca & cb & c^2+1\end{vmatrix} = 1+a^2+b^2+c^2$ **(4)**

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