SECOND PUC PREPARATORY EXAMINATION, MARCH - 2022

SUBJECT : MATHEMATICS (35)

Time : 3 Hrs. 15 Mins. **Max Marks : 100**

INSTRUCTIONS :

i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii) Use the graph sheet for the question on Linear Programming in Part-E

PART - A

I Answer any TEN questions : **10x1=10**

1) The relation R = {(1, 2), (2, 1), (1, 3)} on the set A = {1, 2, 3} is not symmetric, why ?

2) The binary operation $*$ is defined on 'Q' as $a*b=\frac{ab}{2}$, then find $5*3$.

3) Write the domain of $\cot^{-1}(x)$.

4) Find the value of $\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right), 1\times 1\geq 1$

5) Define a diagonal matrix.

6) If A is an invertible matrix of order 2 then find $|A^{-1}|$.

7) If y = log (sin x), then find $\frac{dy}{dx}$.

8) If y = tan (2x + 3), then find $\frac{dy}{dx}$.

9) Find $\int \operatorname{cosec} x\ (\operatorname{cosec} x+\cot x)\cdot dx$.

10) Evaluate : $\int_4^5 e^x\cdot dx$.

11) Compute the magnitude of the vector $\vec{c}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$.

12) Given that $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$, what can you conclude about the vectors $\vec{a}$ and $\vec{b}$?

13) Find the direction cosines of y-axis.

14) Define objective function in a linear programming problem.

15) If $P(A)=\frac{4}{5}$ and $P(B/A)=\frac{2}{5}$ find $P(A\cap B)$.

PART -B

II Answer any TEN questions : **10x2=20**

16) Find gof and fog, if f : R→R and g : R→R are given by f(x) = cos x and g(x) = $3x^2$.

17) Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$.

18) Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\sin^{-1}x, -\frac{1}{\sqrt{2}}\leq x\leq\frac{1}{\sqrt{2}}$

19) If $A=\begin{bmatrix}3 & -1 & 3\\ -1 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & -3\\ 1 & 0\\ 3 & 1\end{bmatrix}$ then find matrix AB.

20) Find the area of the triangle with vertices (1, 0), (6, 0) and (4, 3), using determinants.

(P.T.O.)

21) Find $\frac{dy}{dx}$, if 2x + 3y = sin x.

22) Find the derivative of cos (log x + e^x), x > 0 with respect to x.

23) Differentiate $\cos^{-1}(\sin x)$ with respect to x.

24) Find the approximate value of $\sqrt{25 \cdot 3}$ by using differentials.

25) Find $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

26) Integrate $\frac{1}{x+x\log x}$ with respect to x.

27) Evaluate $\int x \sin x \cdot dx$

28) Find the order and degree of the differential equation y''' + 2y'' + y' = 0.

29) Find a vector in the direction of the vector $5\hat{i}-\hat{j}+2\hat{k}$, which has magnitude 8 units.

30) Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

31) Find the distance of the point (3, -2, 1) from the plane 2x - y + 2z + 3 = 0.

32) Find the vector equation of the plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

33) A random variable X has the following probabilities distribution

X	0	1	2	3	4
P(X)	0.1	k	2k	2k	k

i) determine k ii) p(x = 2).

PART - C

III Answer any TEN questions : **10x3=30**

34) Show that the relation R in the set of $A=\{x : x \in Z \text{ and } 0 \le x \le 12\}$ given by R = {(a, b) : |a - b| is a multiple of 4} is an equivalence relation.

35) Solve $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$.

36) For any square matrix A with real number entries, show that i) A + A' is a symmetric matrix ii) A - A' is a skew symmetric matrix.

37) For the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, verify that, if any two rows of it are interchanged then the sign of the determinant changes.

38) If $x=a(\theta-\sin\theta)$ and $y=a(1+\cos\theta)$ find $\frac{dy}{dx}$.

39) Find the derivative of the function $y^x = x^y$ with respect to x.

40) Verify mean value theorem for the function $f(x) = x^2$ in the interval [2, 4].

41) Find the intervals in which the function $f(x) = x^2 + 2x - 5$ is

i) Strictly increasing ii) Strictly decreasing.

42) Evaluate $\int \frac{x}{(x+1)(x+2)} dx$.

43) Integrate $\frac{(x-3)e^x}{(x-1)^3}$ with respect to x.

44) Find $\int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx$

45) Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the Y-axis, in the first quadrant.

46) Form the differential equation representing the family of curves $y^2 = a(b^2 - x^2)$.

47) Find the general solution of $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$.

48) Find the area of the triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

49) Prove that : $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

50) Find the equation of the plane through the inersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1).

51) A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART - D

IV Answer any SIX questions : **6x5=30**

52) The function, $f: R \rightarrow R$ defined by $f(x) = 1 + x^2$. Verify whether 'f' is one-one, on-to or bijective. Justify your answer.

53) Consider $f: R \rightarrow R$ given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

54) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute (A + B) and (B - C). Also verify that A + (B - C) = (A + B) - C.

55) Solve the following system of linear equations by matrix method :
$x - y + z = 4$; $2x + y - 3z = 0$; $x + y + z = 2$.

56) If $y = \left(\tan^{-1} x\right)^2$, then show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$.

57) A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 2 cm/sec. How fast is its height on the wall decreasing, when the foot of the ladder is 4 m away from the wall ?

58) Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and hence evaluate $\int \frac{dx}{\sqrt{2x - x^2}}$.

59) Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

60) Solve : $x\frac{dy}{dx} + y = x^3, x > 0$.

61) Derive the equation of the line passing through the given point and parallel to a given vector both in Vector and Cartesian form.

62) Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that
i) all the five cards are spades.
ii) none is a spade.

63) Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that
i) the problem is solved.
ii) exactly one of them solves the problem.

PART - E

V Answer any ONE question : **1x10=10**

64) a) Minimise and maximise $Z = -3x + 4y$ subject to the constraints
$x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,\ y \geq 0$.

b) Find k if $f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$.

65) a) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$ and hence find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}} \cdot dx$.

b) If the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, satisfies the equation $A^2 - 4A + I = 0$, where I is 2×2 identity matrix and '0' is a 2×2 zero matrix, then find A^{-1}.

66) a) Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

b) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$

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